

Kumamoto Journal of Science

Series A (Mathematics, Physics and Chemistry)

Vol. 1, No. 2, March 1953

Published by the
Faculty of Science, Kumamoto University
Kumamoto, Japan

熊本大學理學部紀要 第一部 第一卷 第二號 昭和二十八年三月

熊本大學理學部

Kumamoto Journal of Science

Series A: Mathematics, Physics and Chemistry

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ON THE PROJECTIVELY CONNECTED SPACES WITH HOMOGENEOUS COORDINATES WHOSE GROUPS OF HOLONOMY FIX A HYPERQUADRIC.

Yasuo NASU.

(Received Oct. 30, 1952)

This paper is concerned with an n -dimensional projectively connected space H_n with homogeneous coordinates whose group of holonomy fixes a non degenerate hyperquadric Q_{n-1} . For the case of ordinary projectively connected spaces, S. Sasaki, K. Yano and T. Ōtsuki have obtained interesting results.

In an n -dimensional projectively connected space H_n with homogeneous coordinates (x^0, \dots, x^n) , a point x^λ is expressed by

$$x^\lambda = c^\lambda t \quad (\lambda, \mu, \dots = 0, 1, \dots, n; c^\lambda = \text{const.}),$$

where t is a parameter. We must consider the following coordinate transformations:

$$(0.1) \quad \begin{cases} G: & \bar{x}^\lambda = \bar{x}^\lambda(x^0, \dots, x^n), \\ F: & \bar{x}^\lambda = \rho x^\lambda, \quad \rho(x^0, \dots, x^n) \quad (x^\lambda \neq 0), \end{cases}$$

where \bar{x}^λ are homogeneous analytic functions of the first degree in x^λ , such that the functional determinant is different from zero for all points under consideration, and ρ is an analytic function of degree zero in x^λ . The coefficients of the projective connection $\Pi_{\mu\nu}^\lambda$ are homogeneous analytic functions of degree -1 in x^λ , and, by (0.1), $\Pi_{\mu\nu}^\lambda$ are transformed into

$$(0.2) \quad \begin{cases} G: & \bar{\Pi}_{\mu\nu}^\lambda = \frac{\partial \bar{x}^\lambda}{\partial x^\alpha} \left(\frac{\partial x^\beta}{\partial \bar{x}^\mu} \frac{\partial x^\gamma}{\partial \bar{x}^\nu} \Pi_{\beta\gamma}^\alpha + \frac{\partial^2 x^\alpha}{\partial \bar{x}^\mu \partial \bar{x}^\nu} \right), \\ F: & \bar{\Pi}_{\mu\nu}^\lambda = \rho^{-1} \Pi_{\mu\nu}^\lambda. \end{cases}$$

We restrict ourselves to the following case:

$$\Pi_{\mu\nu}^\lambda = \Pi_{\nu\mu}^\lambda, \quad \Pi_{\mu\nu}^\lambda x^\mu = 0.$$

We also restrict ourselves to projective vectors and tensors such that the laws of transformation in (0.1) are given by:

$$\begin{aligned} G: \quad \bar{u}^\lambda &= \frac{\partial \bar{x}^\lambda}{\partial x^\alpha} u^\alpha, & F: \quad \bar{u}^\lambda &= \rho u^\lambda, \\ G: \quad \bar{v}_\lambda &= \frac{\partial x^\alpha}{\partial \bar{x}^\lambda} v_\alpha, & F: \quad \bar{v}_\lambda &= \rho^{-1} v_\lambda, \end{aligned}$$

and

$$G: \bar{w}^{\lambda_1} \dots \lambda_p \dots \mu_1 \dots \mu_q = \frac{\partial \bar{x}^{\lambda_1}}{\partial x^{\rho_1}} \dots \frac{\partial \bar{x}^{\lambda_p}}{\partial x^{\rho_p}} \frac{\partial x^{\sigma_1}}{\partial \bar{x}^{\mu_1}} \dots \frac{\partial x^{\sigma_q}}{\partial \bar{x}^{\mu_q}} w^{\rho_1} \dots \rho_q \sigma_1 \dots \sigma_q,$$

$$F: \bar{w}^{\lambda_1} \dots \lambda_p \dots \mu_1 \dots \mu_q = \rho^{p-q} w^{\lambda_1} \dots \lambda_p \dots \mu_1 \dots \mu_q.$$

Hereafter we assume that the hyperquadric Q_{n-1} at a tangential point x^λ is given by

$$(0.3) \quad Q_{n-1}: G_{\lambda\mu} X^\lambda X^\mu = 0 \quad (\det |G_{\lambda\mu}| \neq 0, G_{\lambda\mu} = G_{\mu\lambda}),$$

where $G_{\lambda\mu}$, X^λ are a covariant projective tensor, and a contravariant projective vector respectively, and the tangential point x^λ does not lie on Q_{n-1} . Therefore we can assume $G_{\lambda\mu} x^\lambda x^\mu = -1$ without any loss of generality.

Under these conditions we shall investigate the structure of the projectively connected spaces with homogeneous coordinates.

1. We consider the following n equations:

$$(1.1) \quad \xi^i = \xi^i(x^0, x^1, \dots, x^n) \quad (i, j, k, \dots = 1, 2, \dots, n),$$

where ξ^i are homogeneous analytic functions of degree zero in x^λ and we assume that the matrix has rank n .

Then we put:

$$(1.2) \quad E^i_{\cdot\lambda} = \frac{\partial \xi^i}{\partial x^\lambda}.$$

Furthermore we must consider a hyperplane:

$$(1.3) \quad p_\lambda x^\lambda = 0,$$

which does not contain the tangential point x^λ and is used as a plane at infinity. This projective covariant vector p_λ enables us to define the inverse of $(E^i_{\cdot\lambda})$. We define the quantities $E_i^{\cdot\lambda}$, $E_o^{\cdot\lambda}$, $E_{\cdot\lambda}^o$ by means of the equations

$$(1.4) \quad E_{\cdot\lambda}^o = p_\lambda, \quad E_o^{\cdot\lambda} = x^\lambda, \quad E_i^{\cdot\lambda} E_{\cdot\lambda}^j = \delta_i^j,$$

$$E_{\cdot\lambda}^i E_i^{\cdot\mu} = \delta_\lambda^\mu - x^\mu p_\lambda, \quad E_i^{\cdot\lambda} p_\lambda = 0, \quad E_{\cdot\lambda}^i x^\lambda = 0.$$

Then we define Γ_{bk}^a ($a, b, c \dots = 0, 1, \dots, n$) as follows:

$$(1.5) \quad \Gamma_{bk}^a = E_{\cdot\lambda}^a E_b^{\cdot\mu} E_k^{\cdot\nu} \Pi_{\mu\nu}^\lambda - E_b^{\cdot\mu} E_k^{\cdot\nu} \frac{\partial}{\partial x^\nu} E_{\cdot\mu}^a.$$

Γ_{bk}^a are analytic functions of degree zero in x^λ , so that we can express as the functions in ξ^i .

Then we get, by putting $a=0, b=0; a=0, b=j; a=i, b=0; a=i, b=j$ in (1.5), the

following equations:

$$(1.6) \quad \begin{aligned} \Gamma_{ok}^o &= 0, & \Gamma_{ok}^i &= \delta_k^i, \\ \Gamma_{jk}^o &= -E_j^\mu E_k^\nu \left(\frac{\partial p^\mu}{\partial x^\nu} - p_\lambda \Pi_{\mu\nu}^\lambda \right), \\ \Gamma_{jk}^i &= E_{\cdot\lambda}^i E_j^\mu E_k^\nu \Pi_{\mu\nu}^\lambda - E_j^\mu E_k^\nu \frac{\partial^2 \xi^i}{\partial x^\mu \partial x^\nu}. \end{aligned}$$

We can easily prove that Γ_{jk}^i are the coefficients of the affine connection, and Γ_{jk}^o are tensor components.

If we define H_{ab} by

$$H_{ab} = E_a^\lambda E_b^\mu G_{\lambda\mu},$$

we can find that $\det |H_{ab}| \neq 0$ in virtue of $\det |G_{\lambda\mu}| \neq 0$.

The covariant differentials ΔH_{ab} with respect to Γ_{bk}^a are related to the covariant differentials $DG_{\lambda\mu}$ with the following equations

$$(1.7) \quad \Delta H_{ab} = E_a^\lambda E_b^\mu (DG_{\lambda\mu} - 2 G_{\lambda\mu} p_\rho dx^\rho),$$

where $\Delta H_{ab} = dH_{ab} - \Gamma_{ak}^c H_{bc} d\xi^k - \Gamma_{bk}^c H_{ac} d\xi^k$, and $DG_{\lambda\mu} = dG_{\lambda\mu} - \Pi_{\lambda\alpha}^\beta G_{\beta\mu} dx^\alpha - \Pi_{\mu\alpha}^\beta G_{\lambda\beta} dx^\alpha$.

If the group of holonomy fixes Q_{n-1} , then $D(G_{\lambda\mu} X^\lambda X^\mu)$ must be proportional to $G_{\lambda\mu} X^\lambda X^\mu$ in virtue of the relation $DX^\lambda = 0$. But as X^λ is an arbitrary vector, we can write these results into

$$(1.8) \quad DG_{\lambda\mu} = (\tau_\rho dx^\rho) G_{\lambda\mu}.$$

Hence we get by putting (1.8) in (1.7) the following relation:

$$\Delta H_{ab} = E_a^\lambda E_b^\mu (\varphi_\rho dx^\rho) DG_{\lambda\mu},$$

where $\varphi_\rho = \tau_\rho - 2p_\rho$. Hence we get:

$$\left(\frac{\partial H_{ab}}{\partial \xi^k} - \Gamma_{ak}^c H_{cb} - \Gamma_{bk}^c H_{ac} \right) E_{\cdot\rho}^k = H_{ab} \varphi_\rho.$$

Therefore

$$(1.9) \quad \frac{\partial H_{ab}}{\partial \xi^k} - \Gamma_{ak}^c H_{cb} - \Gamma_{bk}^c H_{ac} = \varphi_k H_{ab},$$

where $\varphi_k = E_k^\lambda \varphi_\lambda$. If we write (1.9) in full detail, then we get

$$\begin{aligned}
 (1.10) \quad & a = 0, \quad b = 0 \quad : \quad \varphi_k = 2 H_k \quad (H_k = H_{ok} = H_{ko}), \\
 & a = 0, \quad b = j \quad : \quad H_{j;k} + \Gamma_{jk}^o - H_{jk} = 2 H_j H_k, \\
 & a = i, \quad b = j \quad : \quad H_{ij;k} - H_i \Gamma_{jk}^o - H_j \Gamma_{ik}^o = 2 H_k H_{ij},
 \end{aligned}$$

where $H_{j;k} = \frac{\partial H_j}{\partial \xi^k} - \Gamma_{jk}^i H_i$, $H_{ij;k} = \frac{\partial H_{ij}}{\partial \xi^k} - \Gamma_{ik}^l H_{jl} - \Gamma_{jk}^l H_{il}$.

We know, from the definition of φ_ρ , $\varphi_\rho x^\rho = 0$, hence if we replace p_λ with $\bar{p}_\lambda = p_\lambda + \frac{1}{2} \varphi_{\lambda_j}$, then we get from the definitions of E_a^λ , E_b^λ , and Γ_{bk}^a the following relations

$$\begin{aligned}
 \bar{E}_{\cdot\lambda}^i &= E_{\cdot\lambda}^i, & \bar{E}_o^\lambda &= E_o^\lambda, \\
 \bar{E}_j^\mu &= E_j^\mu - x^\mu H_j,
 \end{aligned}$$

and

$$(1.11) \quad \begin{cases} \bar{\Gamma}_{jk}^i = \Gamma_{jk}^i - H_j \delta_k^i - H_k \delta_j^i, \\ \bar{\Gamma}_{jk}^o = \Gamma_{jk}^o - \frac{\partial H_j}{\partial \xi^k} + H_i \Gamma_{jk}^i - H_j H_k. \end{cases}$$

Therefore we get from (1.10), (1.11) the following results.

$$\begin{aligned}
 (1.12) \quad & \bar{\Gamma}_{jk}^o = g_{jk}, \\
 & \bar{\Gamma}_{jk}^i = \{_{jk}^i\},
 \end{aligned}$$

where $g_{ij} = H_{ij} - H_i H_j$ and $\{_{jk}^i\}$ are the Christoffel's symbols. We can easily see $\det |g_{ij}| \neq 0$ from $\det |G_{\lambda\mu}| \neq 0$ (or $\det |H_{ab}| \neq 0$)

Q_{n-1} is also written by putting $Z^a = E_{\cdot\lambda}^a X^\lambda$ as $H_{ab} Z^a Z^b = 0$, and hence we obtain

$$(Z^o)^2 = g_{ij} Z^i Z^j$$

as an equation of Q_{n-1} .

Hereafter we assume that $g_{ij} Z^i Z^j$ is positive definite or negative definite.

Now we can formulate the above mentioned facts as follows:

Theorem I. When the group of holonomy of a projectively connected space with homogeneous coordinates fixes a non degenerate hyperquadric Q_{n-1} , the coefficients of the connection Γ_{ik}^j are induced from $\Pi_{\mu\mu}^\lambda$ as the Christoffel's symbols with respect to g_{ij} which are derived from $G_{\lambda\mu}$.

2. When the paths in the space are given by

$$(2.1) \quad x^\lambda = x^\lambda(u^0, u^1),$$

these equations are satisfied with the following differential equations:

$$(2.2) \quad \frac{\partial^2 x^\lambda}{\partial u^\alpha \partial u^\beta} + \Pi_{\mu\nu}^\lambda \frac{\partial x^\mu}{\partial x^\alpha} \frac{\partial x^\nu}{\partial x^\beta} = \Gamma_{\alpha\beta}^\gamma \frac{\partial x^\lambda}{\partial u^\gamma} \quad (\alpha, \beta, \gamma, \dots = 0, 1).$$

We can consider that $x^\lambda(u^0, u^1)$ are homogeneous analytic functions of first degree in u^α . These differential equations of paths were defined by D. van Dantig. We can easily find that under the transformation of the parameter u^α , $\Gamma_{\alpha\beta}^\gamma$ are transformed like coefficients of projective connexion. Moreover we can see from (2.2) $\Gamma_{\alpha\beta}^\gamma = \Gamma_{\beta\alpha}^\gamma$, $\Gamma_{\alpha\beta}^\gamma u^\alpha = 0$.

According to J. Hantjes, under a suitable transformation of the coefficients of the projective connexion

$$\bar{\Pi}_{\mu\nu}^\lambda = \Pi_{\mu\nu}^\lambda + \delta_\mu^\lambda \varphi_\nu + \delta_\nu^\lambda \varphi_\mu + \varphi_{\mu\nu} x^\lambda,$$

where $\varphi_\mu x^\mu = 0$, $\varphi^\mu + \varphi_{\nu\mu} x^\nu = 0$, it is possible to make the contracted curvature tensor $\bar{\Pi}_{\nu\mu}^\lambda$ with respect to $\bar{\Pi}_{\mu\nu}^\lambda$ identically zero. Moreover he proved that the curvature tensor $R_{\beta\gamma\delta}^\alpha$ with respect to $\Gamma_{\alpha\beta}^\gamma$ is identically zero. Therefore the differential equations (2.2) are reduced to the following form:

$$(2.3) \quad \frac{\partial^2 x^\lambda}{\partial u^\alpha \partial u^\beta} + \Pi_{\mu\nu}^\lambda \frac{\partial x^\mu}{\partial u^\alpha} \frac{\partial x^\nu}{\partial u^\beta} = 0.$$

We know by simple calculation that (2.3) are reduced to the following differential equations:

$$(2.3)' \quad \frac{d^2 x^\lambda}{dp^2} + \Pi_{\mu\nu}^\lambda(1, p) \frac{dx^\mu}{dp} \frac{dx^\nu}{dp} = 0,$$

where $p = u^1/u^0$. Furthermore, by (1.1), we can transform (2.3)' into

$$\frac{d^2 \xi^i}{dp^2} + \bar{\Gamma}_{jk}^i \frac{d\xi^j}{dp} \frac{d\xi^k}{dp} = -2 \left(p_\lambda \frac{dx^\lambda}{ds} \right) \frac{d\xi^i}{dp}.$$

Therefore if we transform the parameter p into s by the equation

$$(2.4) \quad 2 p_\lambda \frac{dx^\lambda}{dp} = \frac{\frac{d^2 p}{ds^2}}{\frac{dp}{ds}},$$

we get

$$(2.5) \quad \frac{d^2 \xi^i}{ds^2} + \bar{\Gamma}_{jk}^i \frac{d\xi^j}{ds} \frac{d\xi^k}{ds} = 0.$$

We can see from (2.5) that s is an affine parameter. Moreover we get from (1.6) the following equations:

$$-E_{\cdot\mu}^j E_{\cdot\nu}^k \Gamma_{jk}^o = \frac{\partial p_{\cdot\mu}}{\partial x^{\nu}} - p_{\lambda} \Pi_{\mu\nu}^{\lambda} + p_{\mu} p_{\nu}.$$

Therefore the above equations are reduced, by differentiating (2.4) with respect to s , to the following equation:

$$(2.7) \quad \{p, s\} = -2 \Gamma_{jk}^o \frac{d\xi^j}{ds} \frac{d\xi^k}{ds},$$

where $\{p, s\}$ is the Schwarzian derivative. From this we conclude that p is a projective parameter.

On the other hand, we can find that the curvature tensor $\Pi_{\cdot\mu\nu\omega}^{\lambda}$ with respect to $\Pi_{\mu\nu}^{\lambda}$ is related by the following relation to the curvature tensor $\bar{R}_{\cdot jkh}^i$ with respect to $\bar{\Gamma}_{jk}^i$.

$$(2.8) \quad E_{\cdot\lambda}^i E_j^{\cdot\mu} E_k^{\cdot\nu} E_h^{\cdot\omega} \Pi_{\cdot\mu\nu\omega}^{\lambda} \\ = \bar{R}_{\cdot jkh}^i + \bar{\Gamma}_{jk}^o \delta_h^i - \bar{\Gamma}_{jh}^o \delta_k^i - \delta_j^i (\bar{\Gamma}_{kh}^o - \bar{\Gamma}_{hk}^o)$$

Since $\Pi_{\mu\nu}^{\lambda} (= \Pi_{\cdot\mu\gamma\lambda}^{\lambda}) = 0$, we get from (2.8) the following relation:

$$\bar{\Gamma}_{jk}^o = -\frac{1}{n-1} R_{jk} \quad (R_{jk} = R_{\cdot jki}^i).$$

Hence from (1.12) the following relation holds good.

$$(2.9) \quad R_{jk} = -(n-1) g_{jk}.$$

We can formulate the above results as follows.

Theorem 2. When the group holonomy of a projectively connected space with homogeneous coordinate fixes a non degenerate hyperquadric, this space is a projectively connected space with corresponding paths including an Einstein space with non vanishing scalar curvature.

3. The differential equations of a path in the Riemann space with the fundamental tensor g_{ij} are given by

$$\frac{d^2 \xi^i}{d\bar{s}^2} + \bar{\Gamma}_{jk}^i \frac{d\xi^j}{d\bar{s}} \frac{d\xi^k}{d\bar{s}} = 0,$$

where the parameter \bar{s} is the arc-length of the path, and is an affine parameter.

This facts imply that there is a relation between \bar{s} and s in (2.5) such that

$$(3.1) \quad \bar{s} = as + b \quad (a \neq 0),$$

where a, b are constants.

Hence we know from (2.7), (2.9), (3.1) the following result:

$$(3.2) \quad \{p, s\} = K$$

where K is a constant and is negative or positive in accordance with g_{ij} being a positive definite or a negative definite tensor.

i) $K < 0$. If we put $K = -2k^2$, then we find the following solution from (3.2)

$$(3.3) \quad p = \frac{m_1 e^{ks} + m_2 e^{-ks}}{n_1 e^{ks} + n_2 e^{-ks}},$$

where m_1, m_2, n_1, n_2 are arbitrary constants with $m_1 n_2 - m_2 n_1 \neq 0$.

Hence we get from (3.3)

$$(3.4) \quad s = \frac{1}{2k} \log \left(\frac{p-p_1}{p_2-p} : \frac{p_0-p_1}{p_2-p_0} \right),$$

where $p_1 = \frac{m_2}{n_2}$, $p_2 = \frac{m_1}{n_1}$ and p_0 are constants such that $\frac{p_0-p_1}{p_2-p_0} = \frac{n_1}{n_2}$.

But by the assumption the tangential point x^λ does not lie on Q_{n-1} . Accordingly we can not apply the Klein's representation of non Euclidean geometry.

2) $K > 0$. By putting $K = 2k^2$ we get similarly

$$s = \frac{1}{2ik} \log \left(\frac{p-p_1}{p_2-p} : \frac{p_0-p_1}{p_2-p_0} \right) \quad (i = \sqrt{-1}).$$

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ON NORMAL COORDINATES IN PROJECTIVELY CONNECTED SPACES WITH HOMOGENEOUS COORDINATES.

Yasuo NASU.

(Received Oct. 30, 1952)

This paper deals with normal coordinates in projectively connected spaces with homogeneous coordinates. O. Veblen, J. M. Thomas and L. P. Eisenhart extended the normal coordinates in Riemann spaces to affinely connected spaces and projectively connected spaces.

On the other hand, J. A. Schouten and J. Hantjes defined the normal coordinates in projectively connected spaces with homogeneous coordinates.

This paper is concerned with the normal coordinates in their papers.

1. We assume that the coefficients of the connexion are given by $\Pi_{\mu\nu}^{\lambda}$ and the coordinate transformations

$$(1.1) \quad \begin{aligned} x^{\lambda'} &= x^{\lambda'}(x^0, \dots, x^n), \\ x^{\lambda'} &= \rho x^{\lambda} \quad (\rho \neq 0), \quad (\lambda', \mu', \nu' \dots = 0, 1, \dots, n) \end{aligned}$$

transform $\Pi_{\mu\nu}^{\lambda}$ into $\Pi_{\mu'\nu'}^{\lambda'}$ as follows:

$$(1.2) \quad \begin{aligned} \Pi_{\beta'\gamma'}^{\alpha'} A_{\alpha'}^{\lambda'} &= \Pi_{\mu\nu}^{\lambda} A_{\beta'}^{\mu} A_{\gamma'}^{\nu} + \partial_{\beta'} A_{\gamma'}^{\lambda} \quad \left(A_{\mu'}^{\lambda} = \frac{\partial x^{\lambda}}{\partial x^{\mu'}} \right), \\ \Pi_{\mu'\nu'}^{\lambda'} &= \rho^{-1} \Pi_{\mu\nu}^{\lambda}, \end{aligned}$$

respectively, where $x^{\lambda'}$ are analytic functions of the first degree in x^{λ} , and $\det |A_{\mu'}^{\lambda}| \neq 0$.

D. van. Dantig defined the differential equations of paths as follows:

$$(1.3) \quad \frac{\partial^2 x^{\lambda}}{\partial u^a \partial u^b} + \Pi_{\mu\nu}^{\lambda} \frac{\partial x^{\mu}}{\partial u^a} \frac{\partial x^{\nu}}{\partial u^b} = \Gamma_{ab}^c \frac{\partial x^{\lambda}}{\partial u^c}.$$

Solutions $x^{\lambda}(u^0, u^1)$ of this differential equations are regarded as analytic functions of the first degree in u^0, u^1 .

For convenience we assume that $\Pi_{\mu\nu}^{\lambda}$ are symmetric with respect to μ and ν . For even if $\Pi_{\mu\nu}^{\lambda} \neq \Pi_{\nu\mu}^{\lambda}$, we can obtain the same results as the symmetric case.

If we put

$$(1.4) \quad t = u^1/u^0,$$

we have, for arbitrary solutions $x^{\lambda}(u^0, u^1)$, the following relations:

$$(1.5) \quad \begin{cases} \frac{\partial x^\lambda}{\partial u^0} = \bar{x}^\lambda - \frac{u^1}{u^0} \frac{d\bar{x}^\lambda}{dt}, & \frac{\partial x^\lambda}{\partial u^1} = \frac{d\bar{x}^\lambda}{dt}, \\ \frac{\partial^2 x^\lambda}{\partial u^{0^2}} = \frac{u^{1^2}}{u^{0^3}} \frac{d^2 \bar{x}^\lambda}{dt^2}, & \frac{\partial^2 x^\lambda}{\partial u^0 \partial u^1} = -\frac{u^1}{u^{0^2}} \frac{d^2 \bar{x}^\lambda}{dt^2}, \\ \frac{\partial^2 x^\lambda}{\partial u^{1^2}} = \frac{1}{u^0} \frac{d^2 \bar{x}^\lambda}{dt^2}, \end{cases}$$

where $\bar{x}^\lambda = x^\lambda(1, t)$. Therefore, by (1.3) and (1.5), we get the following equations:

$$(1.6) \quad \begin{aligned} \frac{u^{1^2}}{u^{0^3}} \left(\frac{d^2 \bar{x}^\lambda}{dt^2} + \bar{\Pi}^\lambda_{\mu\nu} \frac{d\bar{x}^\mu}{dt} \frac{d\bar{x}^\nu}{dt} \right) &= \frac{1}{u^0} \bar{\Pi}^\lambda_{\mu\nu} \bar{x}^\mu \bar{x}^\nu + 2 \frac{u^1}{u^{0^2}} \bar{\Pi}^\lambda_{\mu\nu} \bar{x}^\mu \frac{d\bar{x}^\nu}{dt} \\ &\quad + \frac{1}{u^0} \bar{\Gamma}^0_{00} \bar{x}^\lambda + \left(-\frac{u^1}{u^{0^2}} \bar{\Gamma}^0_{00} + \frac{1}{u^0} \bar{\Gamma}^1_{00} \right) \frac{d\bar{x}^\lambda}{dt}, \\ \frac{u^1}{u^{0^2}} \left(\frac{d^2 \bar{x}^\lambda}{dt^2} + \bar{\Pi}^\lambda_{\mu\nu} \frac{d\bar{x}^\mu}{dt} \frac{d\bar{x}^\nu}{dt} \right) &= \frac{1}{u^0} \bar{\Pi}^\lambda_{\mu\nu} \bar{x}^\mu \frac{d\bar{x}^\nu}{dt} - \frac{1}{u^0} \bar{\Gamma}^0_{01} \bar{x}^\lambda \\ &\quad - \left(\frac{u^1}{u^{0^2}} \bar{\Gamma}^0_{01} + \frac{1}{u^0} \bar{\Gamma}^0_{01} \right) \frac{d\bar{x}^\lambda}{dt}, \\ \frac{1}{u^0} \left(\frac{d^2 \bar{x}^\lambda}{dt^2} + \bar{\Pi}^\lambda_{\mu\nu} \frac{d\bar{x}^\mu}{dt} \frac{d\bar{x}^\nu}{dt} \right) &= \frac{1}{u^0} \bar{\Gamma}^0_{11} \bar{x}^\lambda \\ &\quad + \left(-\frac{u^1}{u^{0^2}} \bar{\Gamma}^0_{01} + \frac{1}{u^0} \bar{\Gamma}^1_{11} \right) \frac{d\bar{x}^\lambda}{dt}, \end{aligned}$$

where $\bar{\Pi}^\lambda_{\mu\nu} = \Pi^\lambda_{\mu\nu}(1, t)$. and $\bar{\Gamma}^c_{ab} = \Gamma^c_{ab}(1, t)$.

The above three equations contain the common factor $\frac{d^2 \bar{x}^\lambda}{dt^2} + \bar{\Pi}^\lambda_{\mu\nu} \frac{d\bar{x}^\mu}{dt} \frac{d\bar{x}^\nu}{dt}$

in the left hand sides and these relations must be compatible. Therefore we find from the last equations that (1.6)₁, (1.6)₂ ought to be written by the same form as (1.6)₃. Hence we obtain the following necessary condition for the compatibility of (1.6):

$$\bar{\Pi}^\lambda_{\mu\nu} \bar{x}^\mu \frac{d\bar{x}^\nu}{dt} = \sigma \frac{d\bar{x}^\lambda}{dt} + \tau \bar{x}^\lambda.$$

As $\Pi^\lambda_{\mu\nu} x^\mu$ is a tensor, we get from the last equation

$$\Pi^\lambda_{\mu\nu} x^\mu = p_\nu x^\lambda + \sigma \delta^\lambda_\nu,$$

where p_μ is a covariant vector and σ is a scalar. Hence we find that $\Pi^\lambda_{\mu\nu} x^\mu x^\nu = P x^\lambda$ ($P = p_\nu x^\nu + \sigma$), where P is a scalar. Now the above relation is written as

$$(1.7) \quad \Pi^\lambda_{\mu\nu} x^\mu = p_\nu x^\lambda + (P - p_p x^p) \delta^\lambda_\nu.$$

The condition (1.7) is necessary in order that the equations are compatible. Conversely,

suppose that the condition (1.7) is satisfied by $\Pi_{\mu\nu}^\lambda$, then (1.6) can be written as follows:

$$\begin{aligned}
 (1.6) \quad & \frac{d^2 \bar{x}^\lambda}{dt^2} + \bar{\Pi}_{\mu\nu}^\lambda \frac{d\bar{x}^\mu}{dt} \frac{d\bar{x}^\nu}{dt} = \frac{u^{0^2}}{u^{1^2}} \left(\bar{P} + 2 \frac{u^1}{u^0} \bar{p}_\nu \frac{d\bar{x}^\nu}{dt} + \bar{T}_{00}^0 \right) \bar{x}^\lambda \\
 & + \frac{u^{0^2}}{u^{1^2}} \left\{ 2 \frac{u^1}{u^0} \left(\bar{P} - \bar{p}_\rho \bar{x}_\rho \right) + \left(- \frac{u^1}{u^0} \bar{T}_{00}^0 + \bar{T}_{00}^1 \right) \right\} \frac{d\bar{x}^\lambda}{dt}, \\
 & \frac{d^2 \bar{x}^\lambda}{dt^2} + \bar{\Pi}_{\mu\nu}^\lambda \frac{d\bar{x}^\mu}{dt} \frac{d\bar{x}^\nu}{dt} = \frac{u^0}{u^1} \left(\bar{p}_\nu \frac{d\bar{x}^\nu}{dt} - \bar{T}_{01}^0 \right) \bar{x}_\lambda \\
 & + \frac{u^0}{u^1} \left\{ \left(\bar{P} - \bar{p}_\rho \bar{x}_\rho \right) - \left(\frac{u^1}{u^0} \bar{T}_{01}^0 + \bar{T}_{01}^1 \right) \right\} \frac{d\bar{x}^\lambda}{dt}, \\
 & \frac{d^2 \bar{x}^\lambda}{dt^2} + \bar{\Pi}_{\mu\nu}^\lambda \frac{d\bar{x}^\mu}{dt} \frac{d\bar{x}^\nu}{dt} = \bar{T}_{11}^0 \bar{x}_\lambda + \left(- \frac{u^1}{u^0} \bar{T}_{11}^0 + \bar{T}_{11}^1 \right) \frac{d\bar{x}^\lambda}{dt},
 \end{aligned}$$

where Γ_{ab}^c are symmetric with respect to a, b and consist of six elements $\Gamma_{00}^0, \Gamma_{01}^0$ ($= \Gamma_{10}^0$), $\Gamma_{00}^1, \Gamma_{01}^1$ ($= \Gamma_{10}^1$), Γ_{11}^0 , and Γ_{11}^1 . The three equations of (1.6)' should be the same. Hence we get:

$$\begin{aligned}
 & \frac{u^{0^2}}{u^{1^2}} \left(\bar{P} + 2 \frac{u^1}{u^0} \bar{p}_\nu \frac{d\bar{x}^\nu}{dt} + \bar{T}_{00}^0 \right) = \frac{u^0}{u^1} \left(\bar{p}_\nu \frac{d\bar{x}^\nu}{dt} - \bar{T}_{01}^0 \right) = \bar{T}_{11}^0, \\
 & \frac{u^{0^2}}{u^{1^2}} \left\{ 2 \frac{u^1}{u^0} \left(\bar{P} - \bar{p}_\rho \bar{x}_\rho \right) + \left(- \frac{u^1}{u^0} \bar{T}_{00}^0 + \bar{T}_{00}^1 \right) \right\} = \frac{u^1}{u^0} \left\{ \left(\bar{P} - \bar{p}_\rho \bar{x}_\rho \right) \right. \\
 & \quad \left. - \left(\frac{u^1}{u^0} \bar{T}_{01}^0 + \bar{T}_{01}^1 \right) \right\} = \left(- \frac{u^1}{u^0} \bar{T}_{11}^0 + \bar{T}_{11}^1 \right).
 \end{aligned}$$

Therefore arbitrary four elements of Γ_{ab}^c are represented by linear combination of the other two. Hence we obtain the following

Theorem. *In a projectively connected space with homogeneous coordinates, the path equations (1.3) can be represented by*

$$(1.8) \quad \frac{d^2 x^\lambda}{dt^2} + \Pi_{\mu\nu}^\lambda \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} = \alpha \frac{dx^\lambda}{dt} + \beta x^\lambda,$$

if and only if the coefficients of the connexion satisfies (1.7), where α, β are arbitrary functions of t .

In the above argument we assumed the symmetry of $\Pi_{\mu\nu}^\lambda$. If we remove this assumption, then (1.6) and (1.6)' will be replaced by four equations.

If Γ_{ab}^c are not symmetric with respect to a, b , then these consist of eight elements $\Gamma_{00}^0, \Gamma_{10}^0, \Gamma_{01}^0, \Gamma_{01}^1, \Gamma_{10}^1, \Gamma_{11}^0, \Gamma_{00}^1$, and Γ_{11}^1 . Therefore arbitrary six elements of Γ_{ab}^c are represented by linear combination of the other two. Hence the above theorem is applicable to the case: $\Pi_{\nu\mu}^\lambda \neq \Pi_{\mu\nu}^\lambda$. But (1.7) must be replaced by the following relations:

$$(1.7) \quad \begin{aligned} \Pi_{\mu\nu}^{\lambda} x^{\mu} &= p_{\nu} x^{\lambda} + (P - p_{\rho} x^{\rho}) \delta_{\nu}^{\lambda}, \\ \Pi_{\mu\nu}^{\lambda} x^{\nu} &= q_{\mu} x^{\lambda} + (P - q_{\rho} x^{\rho}) \delta_{\mu}^{\lambda}. \end{aligned}$$

2. The path equations (1.8) are transformed by (1.1)₂ into

$$\begin{aligned} \frac{d^2 x^{\lambda}}{dt^2} + \Pi_{\mu\nu}^{\lambda} \frac{dx^{\mu}}{dt} \frac{dx^{\nu}}{dt} &= \bar{\alpha} \frac{dx^{\lambda}}{dt} + \bar{\beta} x^{\lambda}, \\ \text{where } \bar{\alpha} &= \alpha \rho - 2 P \frac{d\rho}{dt} + p_x \frac{dx^x}{dt} \frac{d\rho}{dt} + q_x \frac{dx^x}{dt} - 2 \frac{d\rho}{dt}, \\ \bar{\beta} &= \alpha \frac{d\rho}{dt} + \beta \rho - \rho P - \frac{d^2 \rho}{dt^2} - p_x \frac{dx^x}{dt} - q_x \frac{dx^x}{dt}. \end{aligned}$$

Let ρ be a solution of $\bar{\beta} = 0$, then (1.8) reduces to

$$\frac{d^2 x^{\lambda}}{dt^2} + \Pi_{\mu\nu}^{\lambda} \frac{dx^{\mu}}{dt} \frac{dx^{\nu}}{dt} = \bar{\alpha} \frac{dx^{\lambda}}{dt}.$$

Hence we get as the simplest form of the path equations:

$$(2.1) \quad \frac{d^2 x^{\lambda}}{ds^2} + \Pi_{\mu\nu}^{\lambda} \frac{dx^{\mu}}{ds} \frac{dx^{\nu}}{ds} = 0,$$

taking the parameter s defined by $\frac{ds}{dt} = c \exp \left(\int \bar{\alpha} dt \right)$ ($c = \text{const}$). J. Hantjes and K. Yano proved that under the conditions $\Pi_{\lambda\mu}^{\lambda} = \Pi_{\mu\nu}^{\lambda}$ and $\Pi_{\mu\nu}^{\lambda} x^{\mu} = 0$, s is a projective parameter. In general, s is an affine parameter.

3. The normal coordinates at a point x_0^{λ} are defined as a coordinate system which satisfies the conditions:

$$(3.1) \quad \begin{aligned} \left(\Pi_{(\mu\nu)}^{\lambda} \right)_0 &= \left(\partial_{(\omega} \Pi_{\mu\nu)}^{\lambda} \right)_0 = \left(\partial_{(x} \partial_{\omega} \Pi_{\mu\nu)}^{\lambda} \right)_0 = \cdots = 0, \\ \left(A_{\mu}^{\lambda'} \right)_0 &= \delta_{\mu}^{\lambda'}, \end{aligned}$$

where $()_0$ denote the value at x_0^{λ} .

To get such a coordinate system we consider a coordinate transformation

$$(3.2) \quad x^{\lambda'} = x^{\lambda'}(x^{\mu}).$$

Let $x^{\lambda'}$ be an arbitrary coordinate whose path equations are given by (2.1) and x^{λ} be a normal coordinate, then (3.1)₁ must be valid. Hence under these conditions we can

determine, from (1.2)₁, the values of $\frac{\partial^2 x^{\lambda'}}{\partial x^{\alpha} \partial x^{\beta}}$, $\frac{\partial^3 x^{\lambda'}}{\partial x^{\alpha} \partial x^{\beta} \partial x^{\gamma}}$..., at x_0^{λ} ; namely, by

$$\left(\Pi_{(\beta\gamma)}^{\lambda} \right)_0 = 0, \text{ we have } \left(\frac{\partial^2 x^{\lambda'}}{\partial x^{\alpha} \partial x^{\beta}} \right)_0 = - \left(\Pi_{(\alpha'\beta')}^{\lambda'} \right)_0.$$

Similarly, from $(\partial_{(\gamma} \Pi_{\alpha\beta}^{\lambda})_0 = 0, \dots$, etc., we have

$$\begin{aligned} \left(\frac{\partial^3 x^{\lambda}}{\partial x^{\alpha} \partial x^{\beta} \partial x^{\gamma}} \right)_0 &= - \left(\partial_{(\gamma} \Pi_{\alpha'\beta')}^{\lambda} - \Pi_{\mu'(\beta'}^{\lambda} \Pi_{\alpha'\gamma')}^{\mu'} - \Pi_{(\beta'|\mu'|}^{\lambda} \Pi_{\alpha'\gamma')}^{\mu'} \right)_0, \\ \left(\frac{\partial^4 x^{\lambda}}{\partial x^{\alpha} \partial x^{\beta} \partial x^{\gamma} \partial x^{\delta}} \right)_0 &= - \left(\partial_{(\delta} \partial_{\gamma} \Pi_{\alpha'\beta')}^{\lambda} - \partial_{\rho'} \Pi_{(\alpha'\beta'}^{\lambda} \Pi_{\delta'\gamma')}^{\rho'} - 2 \partial_{(\gamma} \Pi_{|\mu'|}^{\lambda} \Pi_{\beta'}^{\mu'} \Pi_{\alpha'\delta')}^{\rho'} \right. \\ &\quad \left. - 2 \partial_{(\gamma} \Pi_{\alpha'|\mu'|}^{\lambda} \Pi_{\beta'\delta')}^{\mu'} + 2 \Pi_{\alpha'\mu'}^{\lambda} \Pi_{(\alpha'\delta')}^{\mu'} \Pi_{\beta'\gamma')}^{\nu'} \right. \\ &\quad \left. - \Pi_{\alpha'|\mu'|}^{\lambda} \Pi_{\beta'\gamma'\delta'}^{\mu'} - \Pi_{\mu'(\alpha'}^{\lambda} \Pi_{\beta'\gamma'\delta')}^{\mu'} \right)_0, \end{aligned}$$

where $\left(\frac{\partial^3 x^{\lambda}}{\partial x^{\beta} \partial x^{\gamma} \partial x^{\delta}} \right)_0 = - \Pi_{\beta'\gamma'\delta'}^{\lambda}$.

If we put

$$\xi^i = \frac{x^i}{x_0} \quad (i, j, k, \dots = 1, 2, \dots, n),$$

then we can determine from the following formulae the values of $\frac{\partial}{\partial \xi^k} A_{\alpha}^{\mu'}$, $\frac{\partial^2}{\partial \xi^i \partial \xi^k} A_{\alpha}^{\mu'}$, ... at x_0^{λ} .

$$\begin{aligned} \frac{\partial}{\partial \xi^k} A_{\alpha}^{\mu'} &= \left(\partial_k A_{\alpha}^{\mu'} \right) x^0, \\ \frac{\partial^2}{\partial \xi^j \partial \xi^k} A_{\alpha}^{\mu'} &= \left(\partial_j \partial_k A_{\alpha}^{\mu'} \right) x^{0^2}, \\ &\dots \dots \dots \\ \frac{\partial^r}{\partial \xi^{j_1} \dots \partial \xi^{j_r}} A_{\alpha}^{\mu'} &= \left(\partial_{j_1} \dots \partial_{j_r} A_{\alpha}^{\mu'} \right) x^{0^r}. \end{aligned}$$

Hence we can define $A_{\alpha}^{\mu'}$ as follows:

$$(3.4) \quad A_{\alpha}^{\mu'} = \delta_{\alpha}^{\mu'} + \frac{1}{1!} \left(\partial_{\xi^i} A_{\alpha}^{\mu'} \right) \left(\xi^i - \xi_0^i \right) + \dots,$$

where ξ_0^i denote the values of ξ^i at x_0^{λ} . Now we can conclude that when $A_{\alpha}^{\mu'}$ are defined for suitable ranges of $|\xi^i - \xi_0^i|$, the coordinate transformation should be defined by

$$(3.5) \quad x^{\lambda} = A_{\alpha}^{\lambda} x^{\alpha}.$$

It remains to show that $A_{\alpha}^{\mu'}$ have the properties $A_{\alpha}^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^{\alpha}}$. But this is easily seen on account of $(\partial_{\beta} A_{\alpha}^{\lambda})_0 = (\partial_{\alpha} A_{\beta}^{\lambda})_0$, $(\partial_{\gamma} \partial_{\beta} A_{\alpha}^{\mu'})_0 = (\partial_{\gamma} \partial_{\alpha} A_{\beta}^{\mu'})_0, \dots$.

Let $x^{\lambda}(s)$ be a solution of (2.1) and a normal coordinate system, then $x^{\lambda}(s)$ are expanded, for a suitable range of s , in power series, namely,

$$x^{\lambda}(s) = x_0^{\lambda} + \frac{s}{1!} \left(\frac{dx^{\lambda}}{ds} \right)_0 + \dots,$$

where $s = 0$ corresponds to the origin x_0^λ . Then we find from the property of normal coordinates (3.1)₁ that

$$\left(\frac{d^2 x^\lambda}{ds^2}\right)_0 = \left(\frac{d^3 x^\lambda}{ds^3}\right)_0 = \dots = 0.$$

Hence the above power series are expressible by

$$(3.7) \quad x^\lambda(s) = x_0^\lambda + \left(\frac{dx^\lambda}{ds}\right)_0 s.$$

If we put $s = u^1/u^0$ in (3.7) then we get from (1.5)

$$x^\lambda(u^0, u^1) = \left(\frac{\partial x^\lambda}{\partial u^0}\right)_0 u^0 + \left(\frac{\partial x^\lambda}{\partial u^1}\right)_0 u^1,$$

because of $x^\lambda(u^0, u^1) = u^0 x^\lambda(1, s)$. Thus we have the analogous conclusion to affinely (or projectively) connected spaces, namely we get the following theorem.

Theorem. *In a projectively connected space with homogeneous coordinates, when the power series (3.4) converge, we can locally define a normal coordinate system at a given point in terms of which any geodesics through the point is expressible by*

$$x^\lambda(u^0, u^1) = C_0^\lambda u^0 + C_1^\lambda u^1,$$

where C_0^λ, C_1^λ are arbitrary constants.

Furthermore we can prove the following theorem by the quite similar method in Riemann spaces.

Theorem. *In a projectively connected space with homogenous coordinates, under an arbitrary coordinate transformation, normal coordinates at a given point are related by linear transformation with constant coefficients to the other normal coordinates at the same point.*

Specially if the coefficients of the connection $\Pi_{\mu\nu}^\lambda$ are symmetric with respect to μ, ν , then the normal coordinates are also applicable to construct normal tensors and extensions of tensors with the quite similar methods in an affine (or a projective) space with symmetric connection.

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A PROOF OF THE SPECTRAL THEOREM

Siro SASAKI

(Received Nov. 21, 1952)

In this note we shall give a proof of the well known spectral formula $f(H) = \int f(x) dE(x)$ for unbounded operators in the Hilbert space.

1. Let A denote the uniformly-closed, self-adjoint and commutative B^* -algebra, with the unit 1, of continuous operators in a Hilbert space \mathfrak{H} . The spectrum of A is a compact Hausdorff space \mathcal{Q} whose element is a continuous character χ of A , that is, a continuous homomorphism of A in the field of complex numbers: $A \rightarrow \chi(A)$ ($A \in A$).

The following theorem is the result of Stone [1] and Gelfand-Neumark [2].

Theorem 1. For every x, y , there is an uniquely determined Radon measure $\mu_{x,y}(\chi)$ on \mathcal{Q} such that $(Ax, y) = \int_{\mathcal{Q}} \chi(A) d\mu_{x,y}(\chi)$ for all $A \in A$. This measure $\mu_{x,y}$ has the properties: μ_{xy} , depends linearly on x ; $\bar{\mu}_{x,y} = \mu_{y,x}$; $\mu_{x,x} \geq 0$; $\|\mu_{x,y}\| \leq \|x\| \|y\|$; $A(\chi) \equiv \chi(A)$ is a continuous function on \mathcal{Q} , and $d\mu_{Ax,y}(\chi) = A(\chi) d\mu_{x,y}$ for all $A \in A$; a bounded operator T commutes with A if and only if $\mu_{Tx,y} = \mu_{x,Ty}$ for all $x, y \in \mathfrak{H}$.

A complex-valued function $f(\chi)$ on \mathcal{Q} or a subset ω of \mathcal{Q} will be called measurable if it is measurable for every measure $\mu_{x,y}$. From Theorem 1, the following facts [3] are deduced: For every bounded and measurable function $f(\chi)$ there corresponds a bounded operator T_f on \mathfrak{H} with $(T_f x, y) = \int f(\chi) d\mu_{x,y}(\chi)$ for all $x, y \in \mathfrak{H}$. This correspondence has the properties; $f \rightarrow T_f$ is an homomorphism; $T_f = T_f^*$; $\|T_f\| \leq \sup_{\chi} |f(\chi)|$; a bounded operator which commutes with A commutes with every T_f ; if a sequence f_n is uniformly bounded and converges to f for every χ , then T_{f_n} converges strongly to T_f .

Let $\varphi_{\omega}(\chi)$ be the characteristic function of a measurable set ω and put $E(\omega) = T\varphi(\omega)$. $E(\omega)$ is a projection. Now, more generally, we shall prove next lemmas.

Lemma 1. For every function $f(\chi)$ of $\chi \in \mathcal{Q}$, there is an additive and homogeneous operator T_f such that $(T_f x, y) = \int f(\chi) d\mu_{x,y}(\chi)$ for every $x \in \mathfrak{D}_f$ and every $y \in \mathfrak{H}$, where \mathfrak{D}_f is the set of all x such that $\int |f|^2 d\mu_{x,x} < +\infty$. T_f has the following properties:

- (1) $T_{\alpha f} = \alpha T_f$ in \mathfrak{D}_f ;
- (2) $T_{f+g} = T_f + T_g$ in $\mathfrak{D}_f \cap \mathfrak{D}_g$; (3) $T_f^* = T_{\bar{f}}$ in $\mathfrak{D}_f = \mathfrak{D}_{\bar{f}}$;
- (4) $E(\omega) \mathfrak{D}_f \subset \mathfrak{D}_f$ and $T_f E(\omega) = E(\omega) T_f$ in \mathfrak{D}_f ;
- (5) $(T_f x, T_g y) = \int f(\chi) \overline{g(\chi)} d\mu_{x,y}(\chi)$ for every $x \in \mathfrak{D}_f$ and every $y \in \mathfrak{D}_g$;
- (6) if $x \in \mathfrak{D}_f$, we have $T_f x \in \mathfrak{D}_g$ if and only if $x \in \mathfrak{D}_{f \cdot g}$, and, when this condition is

satisfied, $T_g T_f x = T_{f \cdot g} x$ where $f \cdot g(x)$ equals to $f(x)g(x)$ whenever both factors are defined and equals to 0 elsewhere.

Proof. By Theorem 1, \mathfrak{D}_f is a linear subspace of \mathfrak{H} , whence T_f exists and is additive and homogeneous. (1), (2) and (3) follow from Theorem 1. Since $E(\omega)$ commutes with A , we have $E(\omega) \mathfrak{D}_f \subset \mathfrak{D}_f$. If $x \in \mathfrak{D}_f$, then $(T_f E(\omega) x, y) = \int f(z) d\mu_{E(\omega)x, y} = (T_f x, E(\omega) y) = (E(\omega) T_f x, y)$, which proves (4).

Since we have $d\mu_{T_f x, y}(z) = f(z) d\mu_{x, y}(z)$ by (4), (5) is valid.

Furthermore, we have (6) from $\int |g(z)|^2 d\mu_{T_f x, T_f x} = \int |f(z)g(z)|^2 d\mu_{x, x}$.

Lemma 2. If $f(z)$ is a measurable function defined almost everywhere in \mathcal{Q} , then T_f is a closed operator with the domain \mathfrak{D}_f everywhere dense in \mathfrak{H} . The adjoint operator T_f^* exists and is identical with $T_{\bar{f}}$.

Proof. It follows from Lemma 1 that $T_{(|f|+1)} T_{(|f|+1)}^{-1} x = x$ for every $x \in \mathfrak{H}$ and $T_{(|f|+1)}^{-1} T_{(|f|+1)} x = x$ for every $x \in \mathfrak{D}_f$. Therefore, $T_{(|f|+1)}^{-1} = T_{(|f|+1)}^{-1}$. Since $T_{(|f|+1)}^{-1}$ is bounded and measurable, the domain of $T_{(|f|+1)}^{-1}$ is \mathfrak{H} and $T_{(|f|+1)}^{-1}$ is self-adjoint. It follows that $T_{(|f|+1)}$ is also self-adjoint and $\mathfrak{D}_{(|f|+1)}$ is everywhere dense in \mathfrak{H} . Since $\mathfrak{D}_f = \mathfrak{D}_{(|f|+1)}$, \mathfrak{D}_f is everywhere dense. We get $g(z) = f(z) (|f(z)| + 1)^{-1}$. By Lemma 1, it follows that $T_f = T_{(|f|+1)} \cdot T_g = T_g T_{(|f|+1)}$ in \mathfrak{D}_f and $T_g^* = T_{\bar{g}}$. Consequently, $(x, T_f^* y) = (x, T_{\bar{f}} y) = (x, T_{\bar{f}} y)$, which proves that $T_f^* = T_{\bar{f}}$ and T_f is closed.

Let λ be a complex number. If a measurable function $f(z)$ is defined almost everywhere in \mathcal{Q} , then the operator $(T_f - \lambda 1)$ exists and is bounded if and only if there exists a positive real number C such that $|f(x) - \lambda| \geq C$ almost everywhere. λ is a characteristic value of T_f if and only if there exists $x \in \mathfrak{H}$ such that $\mu_{x, x}(\omega) > 0$, where ω is the set of z such that $f(z) = \lambda$. If λ is not a characteristic value of T_f , $T_{(f-\lambda)^{-1}}$ is defined and $T_{(f-\lambda)^{-1}} = (T_f - \lambda 1)^{-1}$.

2. We shall denote by A' the set of bounded operators which commute with A , and by $C(A)$ the set of the closed operators which commute with A' .

Lemma 3. If $U \in C(A)$, then

(1) for any x contained in the domain $\mathfrak{D}(U)$ of U , there exists a measurable function $f_x = f_x(z)$ such that $Ux = T_{f_x} x$.

(2) for any x and y contained in $\mathfrak{D}(U)$, there exists a measurable function $f_{x, y} = f_{x, y}(z)$ such that $Ux = T_{f_{x, y}} x$ and $Uy = T_{f_{x, y}} y$.

Proof. (1) Let \mathfrak{M}_x be the set of all $T_f x$ such that $f \in L^2(\mu_{x, x})$; i. e., $\int |f|^2 d\mu_{x, x}(z) < +\infty$. Since \mathfrak{M}_x is isometrically isomorphic to $L^2(\mu_{x, x})$, it is a closed linear subspace. It is clear that $E(\omega) \mathfrak{M}_x \subset \mathfrak{M}_x$, therefore, the projection $P = P(\mathfrak{M}_x)$

commutes with $E(\omega)$, so that $P \in \mathcal{A}'$.

We have $Ux = UPx = PUx \in \mathcal{M}_x$ as we wish to show.

(2) Let $\mathfrak{H} \oplus \mathfrak{H}$ be the direct sum of \mathfrak{H} and \mathfrak{H} , whose elements are the pairs $[u, v]$, where $u \in \mathfrak{H}$ and $v \in \mathfrak{H}$. We denote by $\mathcal{M}_{x,y}$ the subset of all $[T_f x, T_f y] \in \mathfrak{H} \oplus \mathfrak{H}$, such that $f \in L^2(\mu_{x,x} + \mu_{y,y})$. It follows that $\mathcal{M}_{x,y}$ is a closed linear subspace of $\mathfrak{H} \oplus \mathfrak{H}$ and the projection \hat{P} onto $\mathcal{M}_{x,y}$ commutes with $\hat{E}(\omega)$, where $\hat{E}(\omega)$ is a projection defined by $\hat{E}(\omega)[u, v] = [E(\omega)u, E(\omega)v]$. We define P_{11}, P_{12}, P_{21} and P_{22} by $\hat{P}[u, 0] = [P_{11}u, P_{12}u]$ and $\hat{P}[0, v] = [P_{21}v, P_{22}v]$, then bounded operators P_{11}, P_{12}, P_{21} and P_{22} commute with $E(\omega)$. It follows that $[Ux, Uy] = \hat{U}[P_{11}x + P_{21}y, P_{12}x + P_{22}y]$. $\hat{P}[Ux, Uy] \in \mathcal{M}_{x,y}$.

Theorem 2. Let \mathfrak{H} be a separable Hilbert space. For every operator $U \in C(\mathcal{A})$ with everywhere dense domain, there exists a measurable function $f(x)$ such that $U = T_f$: $(Ux, y) = \int f(x) d\mu_{x,y}(x)$ for all $x \in \mathcal{D}(U)$ and all $y \in \mathfrak{H}$.

Proof. By Lemma 3, for any x and y , there exists f_x, f_y and $f_{x,y}$ such that $Ux = T_{f_x}x$, $Uy = T_{f_y}y$, $Ux = T_{f_{x,y}}x$ and $Uy = T_{f_{x,y}}y$. Since T_{f_x}, T_{f_y} and $T_{f_{x,y}}$ commute with \mathcal{A} , we have $f_x(x) = f_{x,y}(x)$ almost everywhere with respect to $\mu_{x,x}$ and $f_y(x) = f_{x,y}(x)$ almost everywhere with respect to $\mu_{y,y}$. By the separability of \mathfrak{H} , there exists an $x \in \mathcal{D}(U)$ such that $\mu_{y,y}$ is absolutely continuous with respect to $\mu_{x,x}$; therefore, for this x , we have $f_x(x) = f_y(x)$ almost everywhere. Since U is closed, $Uy = T_{f_x}y$ for every $y \in \mathcal{D}(U)$. Theorem 2 is thereby proved.

Theorem 3. If H is a self-adjoint operator, we have $H \in C(\mathcal{A})$, where \mathcal{A} is a maximal commutative B^* -algebra which commutes with H , and $(Hx, y) = \int f(x) d(E(x)x, y)$ for all $x \in \mathcal{D}(H)$ and $y \in \mathfrak{H}$.

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ON THE ELECTRICAL CHARACTERISTICS OF CONCRETE AND OTHERS (II)

Shigeichi FUJITA

(Received October 1, 1952)

Abstract

In the 1st report⁽¹⁾, it is shown that the resistances of some concrete are very different when the direction of the electric current is reversed. In this report the results of observation on the details of the phenomena are described. It is supposed that the difference of resistances must be observed when the direction of the electric current through the contact surface of two different materials is changed^{(2) (3) (4)}. The properties of the resistances have been studied by changing the contacting materials under the various conditions and the simple rules have been obtained as to the difference of resistances.

1. The Effect of Gravity

As we have reported in the 1st report, the difference of resistances of concrete which is observed when the current flows in one direction and in the reverse direction seems to have a relation to the direction of gravity. To obtain more definite relation, we have made many concrete columns in erect position, marked α to the upper end and β to the lower. We have applied the voltage 1.5 volts using a dry cell, and the deflections θ_α and θ_β of the galvanometer when the electric currents flow from α to β and from β to α respectively have been read. In this observation, we have obtained a definite result that i_α (the current from α to β) is always greater than i_β (the current from β to α), that is, the resistance R_α (the resistance for i_α) is always smaller than the resistance R_β (the resistance for i_β), when other conditions are the same.

Table I shows the results. In all these specimens we could ascertain easily the fact $R_\alpha < R_\beta$.

Table I.

Specimens	θ_α	θ_β	Specimens	θ_α	θ_β
(32)	23	16	(37)	36	25
(33)	19	11	(38)	57	51
(34)	33	18	(39)	33	26
(35)	18	15	(40)	46	34
(36)	26	17	(41)	26	22

2. The Variation of Current with Time

In the 1st report and 1, it is described of the cases when the readings of the deflection of galvanometer were taken one minute after the voltage has been applied. But the resistances of all the concrete and others varies with the time elapsed after the voltage was first applied. We read, therefore, the deflections of the galvanometer at every 5 seconds or 15 seconds after the application of voltage.

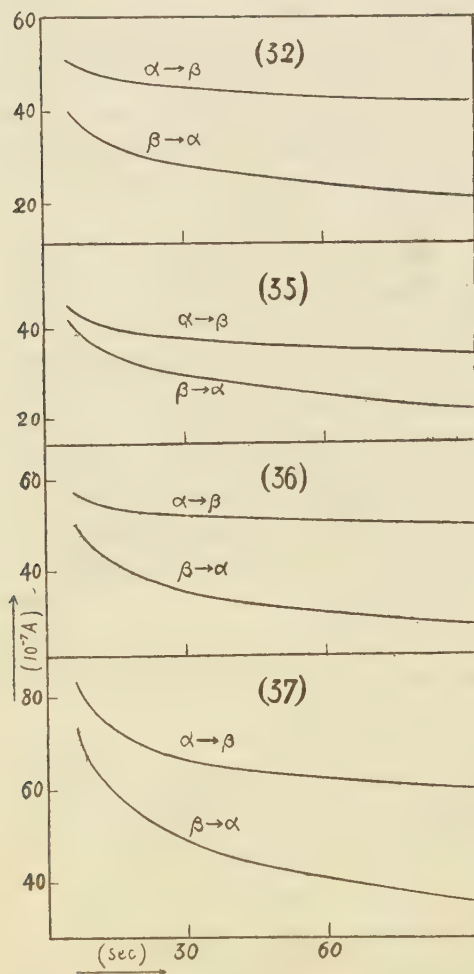


Fig. 1. Variation of currents through concrete columns with time.

Measurements are graphically shown in Fig. 1. Both the currents i_α and i_β becomes smaller and smaller with time, and tends to two different definite values which are different in each concrete column. But the current i_α is always greater than the current i_β . The ratio i_α/i_β slightly increases with time, and tends to a definite value which is different in each concrete column and in different applied voltages.

3. Charge and Discharge of Concrete Columns

When a constant voltage is applied to a concrete column, the current decreases quickly at first but afterwards slowly. It tends to a definite value at end. It is, therefore, supposed that the counter electromotive force is produced as the electric energy is stored in the concrete column⁽⁵⁾. After the charging current becomes almost constant, the battery is cut off, then the counter current flows in the reverse direction for a certain interval caused by the counter E.M.F. produced in the concrete column.

Fig. 2 shows the variations of charging and discharging currents with time. Generally, in the concrete columns which show rapid decay of charging current, the discharging quantity of electricity is large. This fact shows that the decay of charging

current is caused by the counter E.M.F. produced by the charged electricity. More important fact we can notice is that the decay of charging current i_α and the corresponding discharging current are very different from the decay of charging current i_β and the corresponding discharging current.

4. Application of Alternate Voltage

The resistances of the concrete columns and others are different if the initial conditions of the specimens are different. They are charged from beginning by some natural electric field existing in the place where they are. Though the natural electric field applied to them are always very feeble, but it is not negligible and varies with time, so that the measured resistances are different from time to time. Moreover the electrical properties of them depend much upon their histories, that is, they have electric hysteresis⁽⁶⁾.

To investigate the effect of hysteresis, we have applied on them a voltage 1.5 volts in the direction of $\alpha \rightarrow \beta$ (or $\beta \rightarrow \alpha$) during a time interval sufficiently long until the strength of current becomes almost constant. The strength of current i_α (or i_β) is measured at every 15 seconds during the time interval, then immediately afterwards the direction of the voltage is reversed in the sense $\beta \rightarrow \alpha$ (or $\alpha \rightarrow \beta$) and i_β (or i_α) is measured as before during the same time interval, then the direction of voltage is reversed again in $\alpha \rightarrow \beta$ (or $\beta \rightarrow \alpha$). Thus the reversal of voltage is successively repeated several times⁽⁶⁾, during which i_α and i_β are measured at every 15 seconds. The results of measurement are

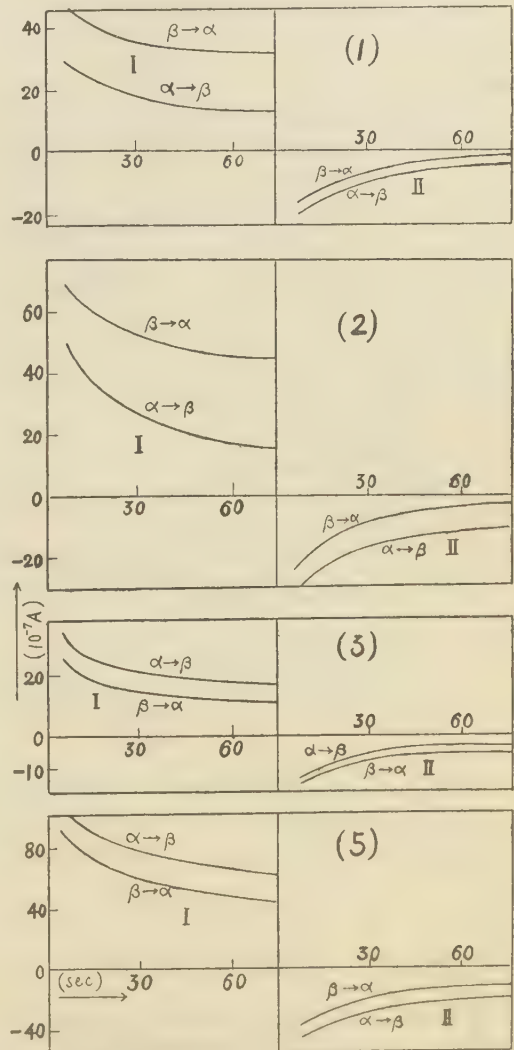


Fig. 2. Variation of charge and discharge currents with time.

The results of measurement are

plotted in the Fig. 3. Suffix of α and β means the order of observation in the repeated reversal of the direction of current. In (61), the upper part of the cube is the concrete composed of equal quantity of sand and cement, the lower part contains no sand, the current i_β is greater than the current i_α .

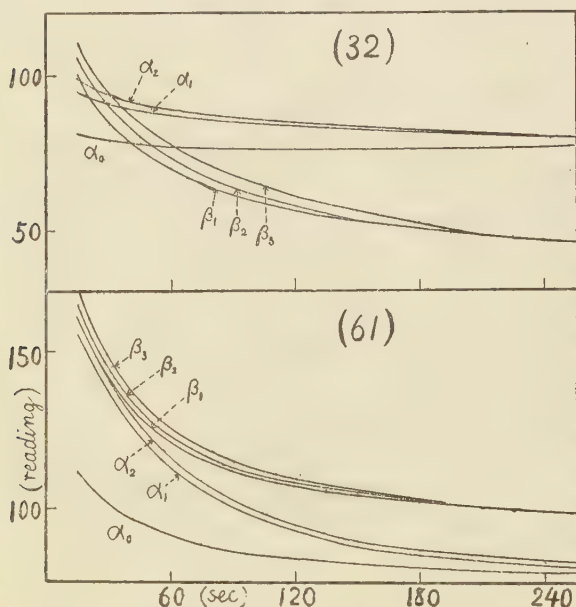


Fig. 3. Currents by the application of alternate voltage.
 α shows the direction of current ($\alpha \rightarrow \beta$).
 β shows the direction of current ($\beta \rightarrow \alpha$).

Almost of all stones have no difference of resistances in two directions ($\alpha \rightarrow \beta$) and ($\beta \rightarrow \alpha$), but sometimes there are pieces of sedimentary rocks which have slight difference of resistances in two directions perpendicular to the layer of the sediment.

5. Concrete Contact with Stones

Since it is considered that the inequality of i_α and i_β in concrete would be caused in the contact surfaces of cement with pebbles, we made experiments to ascertain this consideration by measuring i_α and i_β of concrete columns cemented with various stones in one end of them, expecting the inequality of i_α and i_β . The procedure of measurement is the same as in 4. The direction of a applied constant voltage (1.5 volts or 3 volts) is reversed with

From these curves, it becomes clear that after one period of alternate application of voltage, the variation of the strength of current with time is almost the same in each period. We have, therefore, measured during only 2 or 3 periods, and could obtain characteristic curves peculiar to each specimen expressing the variation of the strength of current with time.

In these curves, it is shown that the variation of i_α with time is different from that of i_β , and the difference is also different in each specimen. If we take i_α and i_β at the same epoch after the direct voltage and the reverse voltage was applied, i_α/i_β is always greater than unity in concrete columns made in erect position as seen in (32).

a time interval during which the currents i_α or i_β becomes almost constant. The variations of i_α and i_β with time were plotted in Fig. 4.

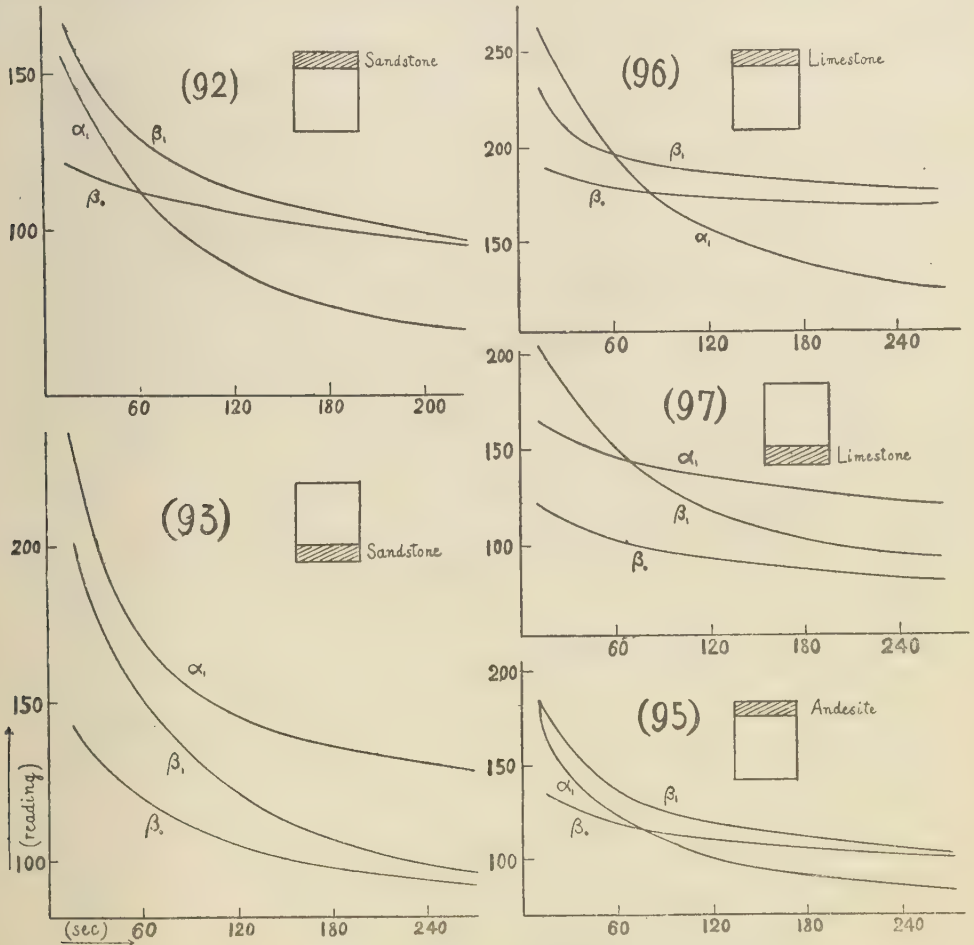


Fig. 4. Currents through concrete columns (α and β) having a piece of stone at one end.

(92), (93), (95), (96) and (97) shows the cases of concrete columns, having a piece of sand stone, lime stone or Shimasaki-ishi (andesite) cemented to upper or lower end of it. In all these cases electric current flows more easily in the direction from cement to stone than in the reverse direction. The difference of resistances in two directions, ($\alpha \rightarrow \beta$) and ($\beta \rightarrow \alpha$), varies with time, and the variation of the difference is peculiar to each stone cemented to concrete.

6. Concrete Contact with Metals

As we have ascertained that the inequality of i_α and i_β in concrete is caused from the

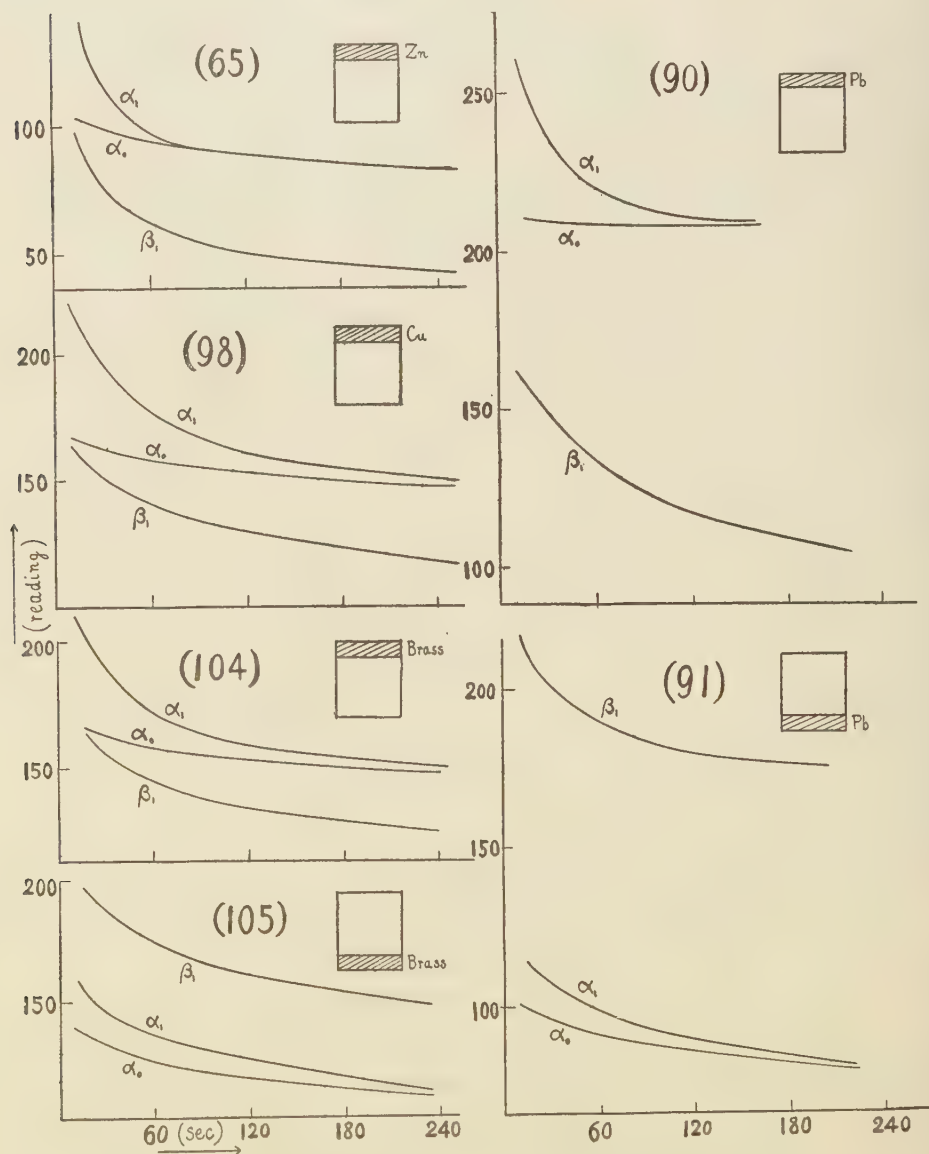


Fig. 5. Currents through concrete columns (α and β) having a metal plate at one end.

contact surface of cement and stones, it was supposed that perhaps the contact surfaces of cement and metals would have the similar electrical property. To examine this consideration, we measured as before the resistances of concrete having a metal plate cemented at an end of it. Some examples of the results of observation is plotted in Fig. 5.

In many of cases of concrete having a metal plate cemented at an end of it, electric

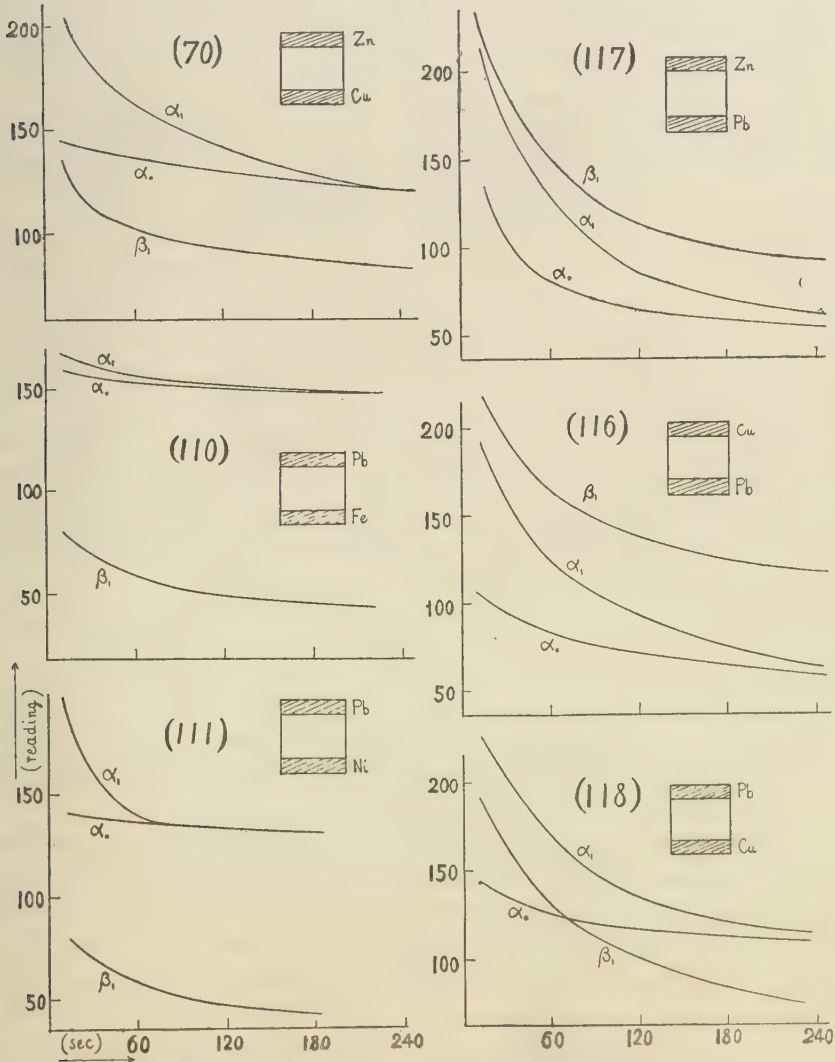


Fig. 6. Currents through concrete columns (α and β) having two metal plates at both ends

current flows more easily from metal to cement than from cement to metal as shown in (65), (90), (91), (98) and (104), and in these cases the differences of resistances in two directions ($\alpha \rightarrow \beta$) and ($\beta \rightarrow \alpha$) are generally large compared to the case of 5. Especially in the case of Lead, the difference is very large compared to Zinc, Copper, Iron, •••. Thus the difference of resistances depends largely on the metals cemented at one end of cement. Accordingly we will be able to arrange the metals in a series by distinguishing this property of metals.

For this purpose, we have made concrete cubes, having a metal plate at upper end and other kind of metal plate at its lower end cemented, and measured i_α and i_β as before. Some examples of this measurement are shown in Fig. 6.

In the concrete (70) having Zn plate at upper end and Cu plate at lower end, the electric current flows more easily in the direction from Zn to Cu than in the reverse direction. In (109) having Al plate at upper end and Ni plate at lower end, the electric current flows more easily in the direction from Al to Ni than in the reverse direction. In (110) and (111) having Pb plate at one end and Fe or Ni plate at the opposite end, the electric current flows more easily in the direction from Pb to Fe or Ni than in the reverse direction. In (116) and (118), it is shown that the electric current flows more easily in the direction from Pb to Cu than in the reverse direction. In (117) it is shown that the electric current flows more easily in the direction from Pb to Zn than in the reverse direction.

The measurements like these would enable us to arrange metals in a series such as Al, Pb, Zn, Cu, Fe, •••, in which if we select two of metals and cement them at both ends of a concrete column, the electric current flows more easily in the direction from left metal to right metal than in the reverse direction. But this order of series would be changed by some changes of conditions of metals and concrete.

7. Conclusion

We have ascertained that in the ordinary concrete column, containing gravel, which were made in erect position, the electric current flows more easily in the direction of gravity at the time of making than in the reverse direction.

We have observed in details the variations of currents with time under the application of a constant voltage in various concrete columns, and ascertained that the resistance of a concrete column having any stone or metal cemented in one end changes also when the direction of electric current is reversed, and that the electric current decays in the manner peculiar to each column, and the decay of current is also different when the direction of voltage is reversed.

It became clear that these differences are caused from the contact surfaces of concrete and stones or metals, and the magnitude of the difference is peculiar to each stone or metal. In most of cases of stones, the current flows more easily in the direction from cement to

stone than in the reverse direction, while in most of cases of metals, the current flows more easily in the direction from metal to cement than in the reverse direction.

In concrete columns which have two different metal plates cemented at both end surfaces, the current flows more easily in the direction from a certain metal plate to another than in the reverse direction. By measuring the strength of current in both directions we can arrange the metals in a series such that any two of the metals are cemented at both ends of a concrete column, the electric current flows more easily in the direction from left one to right one than in the reverse direction.

In conclusion, I wish to express sincere gratitude to Prof. Nakamura and to Prof. Namba for their kind guidance and encouragement and to other colleagues in the laboratory for their discussion and encouragement.

This report was read Dec. 1951 at the regular meeting of Kyushu Branch of the Physical Society of Japan.

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RESEARCH ON THE COMPENSATED PENDULUM

Ryuzo ADACHI

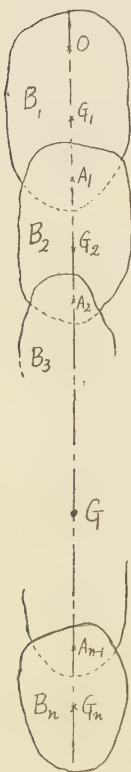
(Received September 20, 1952)

1. Introduction

Suppose a pendulum which is constructed with several bodies, each body is consisted of uniform material, and the period of it is independent of the temperature (so-called compensated pendulum). When we make such a pendulum many conditions among shapes of each body, coefficients of expansion, positions of connecting point etc., must be satisfied.

In the following section, we give general conditions that the period is independent of the temperature and some attentions are mentioned.

2. General theory



Let B_1, B_2, \dots, B_n be bodies and each of them is consisted of uniform material, and m_i, G_i be the mass, the center of gravity of B_i respectively. A pendulum is constructed by connection of B_1, B_2, \dots, B_n . As shown in Fig.1 these bodies are connected with A_1, A_2, \dots, A_{n-1} and $O, G_1, A_1, G_2, A_2, \dots, A_{n-1}, G_n$ lay on a straight line, where O is the supporting point of the pendulum, and G is the center of gravity.

$$\text{Put } \overrightarrow{OG} = h, \overrightarrow{OG_1} = p_1, \overrightarrow{G_1A_1} = q_1, \overrightarrow{A_1G_2} = p_2, \dots, \overrightarrow{A_{n-1}G_n} = p_n, \quad p_i + q_i = r_i$$

and

$$\sum_{i=1}^n m_i = M \dots \dots \dots (1).$$

Let I be the moment of inertia of the pendulum about O and T be the period, then

$$T = 2\pi \sqrt{I/Mgh} \dots \dots \dots (2).$$

and

$$Mh = m_1 p_1 + m_2 (r_1 + p_2) + \dots + m_n (r_1 + \dots + r_{n-1} + p_n) \dots (3).$$

Accordingly if we denote the value of h by h_t when the temperature change is t^0 , we have

$$Mh_t = m_1 p_1 (1 + \alpha_1 t) + m_2 \{r_1 (1 + \alpha_1 t) + p_2 (1 + \alpha_2 t)\} + \dots \dots \dots + m_n \{r_1 (1 + \alpha_1 t) + \dots + r_{n-1} (1 + \alpha_{n-1} t) + p_n (1 + \alpha_n t)\}.$$

Fig. 1.

Therefore putting

$$H = \frac{1}{M} \sum_{i=1}^n m_i (r_1 \alpha_1 + \dots + r_{i-1} \alpha_{i-1} + p_i \alpha_i) \dots \dots \dots (4).$$

we get

$$h_t = h + Ht \dots \dots \dots (5).$$

where α_i is the coefficient of linear expansion of B_i . Next let I_i be the moment of inertia of B_i about G_i , and put $I_i = m_i k_i^2$, then

$$I = (I_1 + m_1 p_1^2) + \{I_2 + m_2 (r_1 + p_2)^2\} + \dots \dots \dots$$

$$\therefore I = \sum_{i=1}^n m_i \{k_i^2 + (r_1 + \dots + r_{i-1} + p_i)^2\} \dots \dots \dots (6).$$

Similarly

$$I_t = \sum_{i=1}^n m_i \left[k_i^2 (1 + \alpha_i t)^2 + \{r_1 (1 + \alpha_1 t) + \dots \dots \dots \right. \\ \left. \dots \dots \dots + r_{i-1} (1 + \alpha_{i-1} t) + p_i (1 + \alpha_i t)\}^2 \right]$$

and neglecting higher order of $\alpha_i t$ and putting

$$J = 2 \sum_{i=1}^n m_i \left[k_i^2 \alpha_i + (r_1 + \dots + r_{i-1} + p_i) (r_1 \alpha_1 + \dots + r_{i-1} \alpha_{i-1} + p_i \alpha_i) \right] \dots (7).$$

we get

$$I_t = I + Jt \dots \dots \dots (8).$$

Accordingly if the period T is independent of the temperature t , we get

$$I_t h_t = (I + Jt)/(h + Ht) = \text{const.}$$

therefore

$$F \equiv IH - hJ = 0 \dots \dots \dots (9).$$

Moreover if the length of the equivalent simple pendulum is l and putting $E = I/M$,

we get

$$\phi \equiv lh - E = 0 \dots \dots \dots (10).$$

Consequently the necessary and sufficient condition that the length of the equivalent simple pendulum is l and the period is independent of the temperature is that the simultaneous equations (9) and (10) are satisfied,

If we assume $r_1, r_2, \dots, r_{n-1}; p_1, p_2, \dots, p_n; m_1, m_2, \dots, m_n; k_1^2, k_2^2, \dots, k_n^2$ as variables, (9) and (10) must be satisfied by these $4n-1$ variables.

3. Small variation of l

For example, a pendulum-clock is abjusted to indicate the accurate time in a place independently of the temperature, and then it has layed in other place where the value of g differs from the previous one. In order to correct this clock to denote accurate time we move the position of the weight, then will the independency of the temperature be satisfied? In the following we consider such problem.

If I, h, H, J, E, l have changed to $I+\delta I, h+\delta h, \dots, l+\delta l$ respectively, and the independency is satisfied, then from (9), (10) we have

$$\begin{aligned}(I+\delta I)(H+\delta H)-(h+\delta h)(J+\delta J) &= 0 \\ (l+\delta l)(h+\delta h)-(E+\delta E) &= 0\end{aligned}$$

that is

$$I\delta H + H\delta I - h\delta J - J\delta h = 0 \quad (11).$$

$$l\delta h + h\delta l - \delta E = 0 \quad (12).$$

Therefore (9), (10), (11) and (12) must be satisfied and these are necessary and sufficient conditions that the period is independent of the temperature when the length of the equivalent simple pendulum is $l+\delta l$.

For example, if I, h, H, J, E depend on a variable x (when a pendulum-clock the displacement of the weight is x), then

$$\delta I = \frac{\partial I}{\partial x} \delta x, \quad \delta h = \frac{\partial h}{\partial x} \delta x, \dots, \quad \delta E = \frac{\partial E}{\partial x} \delta x$$

$$\therefore \left(I \frac{\partial H}{\partial x} + H \frac{\partial I}{\partial x} - h \frac{\partial J}{\partial x} - J \frac{\partial h}{\partial x} \right) \delta x = 0$$

where δx is arbitrary, therefore

$$I \frac{\partial H}{\partial x} + H \frac{\partial I}{\partial x} - h \frac{\partial J}{\partial x} - J \frac{\partial h}{\partial x} = 0 \quad (13).$$

moreover

$$\left(l \frac{\partial h}{\partial x} - \frac{\partial E}{\partial x} \right) \delta x + h\delta l = 0 \quad (14).$$

Consequently, if the pendulum has made so as to be satisfied (9), (10), (13) and we give a change δx to x then l becomes to $l+\delta l$ and the independency of the temperature is satisfied, where δl is determined from (14).

4. When the length is quadratic with respect to the temperature

If we assume that the length of body B_i is

$$(\text{initial length}) \times (1 + \alpha_i t + \beta_i t^2)$$

when the temperature change is t° , from (3) we have

$$h_t = \frac{1}{M} \sum_{i=1}^n m_i \left\{ r_1 (1 + \alpha_1 t + \beta_1 t^2) + \dots + r_{i-1} (1 + \alpha_{i-1} t + \beta_{i-1} t^2) + p_i (1 + \alpha_i t + \beta_i t^2) \right\}$$

and putting

$$L = \frac{1}{M} \sum_{i=1}^n m_i (r_1 \beta_1 + \dots + r_{i-1} \beta_{i-1} + p_i \beta_i) \dots \dots \dots (15).$$

we get

$$h_t = h + Ht + Lt^2 \dots \dots \dots (16).$$

From (6) we have

$$I_t = \sum_{i=1}^n m_i \left[k_i^2 (1 + \alpha_i t + \beta_i t^2)^2 + \left\{ r_1 (1 + \alpha_1 t + \beta_1 t^2) + \dots \dots \dots + r_{i-1} (1 + \alpha_{i-1} t + \beta_{i-1} t^2) + p_i (1 + \alpha_i t + \beta_i t^2) \right\}^2 \right]$$

and putting

$$K = \sum_{i=1}^n m_i \left[k_i^2 (\alpha_i^2 + 2\beta_i) + (r_1 \alpha_1 + \dots + r_{i-1} \alpha_{i-1} + p_i \alpha_i)^2 + 2(r_1 + \dots + r_{i-1} p_i)(r_1 \beta_1 + \dots + r_{i-1} \beta_{i-1} + p_i \beta_i) \right] \dots \dots (17).$$

we get

$$I_t = I + Jt + Kt^2 \dots \dots \dots (18).$$

Therefore if $I_t/h_t = \text{const.}$, we obtain

$$I/h = J/H = K/L \dots \dots \dots (19).$$

Consequently, besides the condition (9), the condition

$$I/h = K/L \dots \dots \dots (20).$$

must be satisfied.

5. When some of m_1, m_2, \dots, m_n are small values

When some of m_1, m_2, \dots, m_n are small as compared with M , and putting them n_1, n_2, \dots, n_s and we consider that I_t, h_t are functions of n_1, n_2, \dots, n_s and t , that is

$$\left. \begin{aligned} I_t &= I_t(n_1, n_2, \dots, n_s; t) \\ h_t &= h_t(n_1, n_2, \dots, n_s; t) \end{aligned} \right\} \dots\dots\dots (21).$$

Further suppose $\partial I_t/\partial t, \partial h_t/\partial t$ are small, and putting

$$\left. \begin{aligned} \Delta h_m &= \sum_{i=1}^s n_i \frac{\partial}{\partial n_i} h_t(0, 0, \dots, 0; 0) \\ \Delta h_t &= \frac{\partial}{\partial t} h_t(0, 0, \dots, 0; 0) \\ h_0 &= h_t(0, 0, \dots, 0; 0) \\ \Delta I_m &= \sum_{i=1}^s n_i \frac{\partial}{\partial n_i} I_t(0, 0, \dots, 0; 0) \\ \Delta I_t &= \frac{\partial}{\partial t} I_t(0, 0, \dots, 0; 0) \\ I_0 &= I_t(0, 0, \dots, 0; 0) \end{aligned} \right\} \dots\dots\dots (22).$$

$$\left. \begin{aligned} \Delta I_m &= \sum_{i=1}^s n_i \frac{\partial}{\partial n_i} I_t(0, 0, \dots, 0; 0) \\ \Delta I_t &= \frac{\partial}{\partial t} I_t(0, 0, \dots, 0; 0) \\ I_0 &= I_t(0, 0, \dots, 0; 0) \end{aligned} \right\} \dots\dots\dots (23).$$

we get

$$\left. \begin{aligned} h_t &= h_0 + \Delta h_m + \Delta h_t t \\ I_t &= I_0 + \Delta I_m + \Delta I_t t \end{aligned} \right\} \dots\dots\dots (24).$$

and from $\partial T/\partial t = 0$ we have $h_t \frac{\partial I_t}{\partial t} - I_t \frac{\partial h_t}{\partial t} = 0$.

Substituting (24) into above relation and neglecting higher order of $\Delta h_m, \Delta h_t, \Delta I_m, \Delta I_t$ we get

$$h_0 \Delta I_t - I_0 \Delta h_t = 0 \dots\dots\dots (25).$$

This relation does not involve n_1, n_2, \dots, n_s , therefore if $\partial T/\partial t = 0$ is satisfied when $n_1 = n_2 = \dots = n_s = 0$, then the condition $\partial T/\partial t = 0$ is satisfied when n_1, n_2, \dots, n_s take small values which are not zero.

This relation gives us very convenient manner when we try to design a pendulum.

6. Slight disturbance of the collinearity of $A_1, A_2, \dots, A_{n-1}; G^1, G_2, \dots, G_n; O$

As shown in Fig. 2 we take $\overrightarrow{OG_1}$ as x -axis and coordinates of G_i, G are denoted by $(x_i, y_i, z_i), (\bar{x}, \bar{y}, \bar{z})$ respectively.

The directioncosines of $\overrightarrow{A_{i-1}G_i} = p_i, \overrightarrow{G_iA_i} = q_i$ are denoted by $\lambda_i, \mu_i, \nu_i; \rho_i,$

σ_i, τ_i respectively. Then there exist following relations

$$\left. \begin{aligned} x_i &= p_1 + q_1 \rho_1 + p_2 \lambda_2 + \dots + p_{i-1} \lambda_{i-1} + q_{i-1} \rho_{i-1} + p_i \lambda_i \\ y_i &= q_1 \sigma_1 + p_2 \mu_2 + \dots + p_{i-1} \mu_{i-1} + q_{i-1} \sigma_{i-1} + p_i \mu_i \\ z_i &= q_1 \tau_1 + p_2 \nu_2 + \dots + p_{i-1} \nu_{i-1} + q_{i-1} \tau_{i-1} + p_i \nu_i \end{aligned} \right\} \dots \dots (26).$$

If we suppose that angles between p_i and x -axis, q_i and x -axis are very small and are infinitesimal of first order, we get

$$x_i = r_1 + \dots + r_{i-1} + p_i$$

and y_i, z_i are first order together.

Therefore from the relation

$$\bar{x} = \frac{1}{M} \sum_{i=1}^n m_i x_i, \quad \bar{y} = \frac{1}{M} \sum_{i=1}^n m_i y_i, \quad \bar{z} = \frac{1}{M} \sum_{i=1}^n m_i z_i$$

h becomes

$$h = \overrightarrow{OG} = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2} = \bar{x}$$

$$\therefore h = \frac{1}{M} \sum_{i=1}^n m_i (r_1 + \dots + r_{i-1} + p_i).$$

Further

$$I = \sum_{i=1}^n m_i (k_i^2 + x_i^2 + y_i^2 + z_i^2) = \sum_{i=1}^n m_i (k_i^2 + x_i^2).$$

Accordingly it is entirely the same that is previously mentioned.

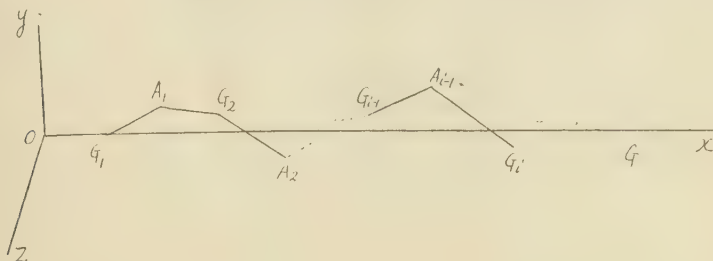


Fig. 2.

7. Examples

At the conclusion of our discussion, we consider simple examples which are as follows.

EX. 1. As in ordinary pendulum-clock we suppose a pendulum which is constructed with band spring B_1 , supporting bar B_2 , and weight B_3 .

By section 5. we assume $m_1 = m_2 = 0$, $I_1 = I_2 = 0$ and putting $\alpha_2/\alpha_1 = \delta$, $\alpha_3/\alpha_1 = \varepsilon$, $r_1 = c$, $r_2 = 2a$, $p_3 = -b$, $k_3^2 = \frac{1}{2}b^2$ we have

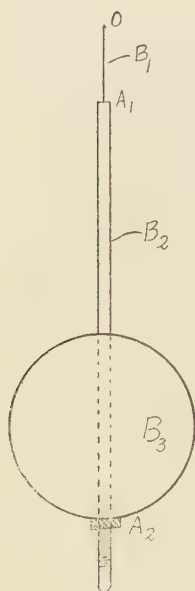


Fig. 3

$$M = m_3, \quad I = M \left(\frac{1}{2} b^2 + h^2 \right), \quad h = c + 2a - b, \quad H = \alpha_1 (c + 2a\delta - b\epsilon), \\ J = M (b^2 \epsilon \alpha_1 + 2Hh).$$

Moreover, we put

$$h/b = x, \quad \epsilon - \frac{1}{2} = \sigma, \quad (\delta - 1) \frac{2a}{b} = e, \quad \epsilon - 1 - e = \rho,$$

then from (9) we get

$$x^3 - \rho x^2 + \sigma x + \frac{1}{2} \rho = 0 \quad (i).$$

Next, from (10) we have

$$x^2 - l x \cdot \frac{1}{b} + \frac{1}{2} = 0 \quad (ii)$$

and usually

$$b < a, \quad 2a - b < h \therefore 1 < \frac{2a}{b} - 1 < x \quad (iii).$$

Therefore if we give ρ, σ (i.e. $\alpha_1: \alpha_2: \alpha_3, 2a/b$ are given) the value of x which is determined from (i) must be greater than 1, and if x is so determined, then from (ii) the value of b (therefore the value of h) is determined. Consequently the necessary and sufficient

condition that the period of this pendulum is independent of the temperature is that the equation (i) has a real root greater than 1.

And by trouble-some calculation this condition becomes

$$e \leq \sigma - \frac{1}{2} - \sqrt{\tau_2} \quad (iv)$$

where τ_2 is determined from following relations

$$J = \sigma^4 + 14\sigma^3 + \frac{135}{2}\sigma^2 + \frac{243}{2}\sigma + \frac{729}{16}$$

$$\tau_2 = \frac{1}{4} \left(\frac{27}{4} + 9\sigma - \sigma^2 + \sqrt{J} \right)$$

e. g.

$$\text{if } \begin{cases} \alpha_1 = 0.13 \times 10^{-3} \\ \alpha_2 = 0.01 \times 10^{-4} \\ \alpha_3 = 0.26 \times 10^{-4} \end{cases} \quad \text{then } \begin{cases} e = -0.923u \\ \rho = 1 + 0.923u \\ \text{where} \\ u = 2a/b \end{cases}$$

and $J = 413, \quad \tau_2 = 9.7.$

Therefore (iv) becomes $-0.923u \leq 1.00 - \sqrt{9.7} = -2.11 \therefore 2.29 \leq u$ and from (i) we get

$$u = \frac{x^3 - x^2 + 1.5x + 0.5}{0.923(x^2 - 0.5)} \dots\dots\dots (v).$$

On the other hand from (iii) we have

$$u - 1 < x \dots\dots\dots (vi)$$

and from (v). (vi) we get following relation

$$x^3 - 25.0x^2 + 25.5x + 12.5 < 0 \dots\dots\dots (vii).$$

If we put the left side of (vii) to $\mathcal{U}(x)$ then the equation $\mathcal{U}(x) = 0$ has three real roots which are in the intervals $(-\infty, 1)$, $(1, 2)$, $(24, 25)$ and we denote these roots by a_1 , a_2 , a_3 respectively, then (vii) becomes

$$(x - a_1)(x - a_2)(x - a_3) < 0$$

$$\therefore a_2 < x < a_3, \text{ that is } 1. \times \times \dots\dots < x < 24. \times \times \dots\dots\dots$$

If we give any value to x in this interval, in obedience to x the value of b is determined and then the pendulum is determined.

$$\text{e.g. if } x = 10.0 \text{ then } a = 5b, \quad h = 10b$$

$$x = 20.0 \dots\dots a = 10.34b, \quad h = 20b$$

and the value of b is determined from (ii).

EX. 2 When a pendulum is constructed with two bodies B_1 , B_2 , where B_1 is a slender bar, B_2 consists of two parts B'_2 , B''_2 and B'_2 is a slender cylinder which is fixed to B_1 , and B''_2 can be displaced along B'_2 .

By section 5. we assume that

$$(\text{mass of } B_1, B_2) = 0 \text{ and put } \alpha_2/\alpha_1 = \delta.$$

Now, let R be a fixed point on B'_2 , and put $\overrightarrow{A_1 R} = y$,

$$G\vec{R} = x$$

then

$$p_2 = y - x, \quad h = r_1 + p_2 = r_1 + y - x$$

$$H = r_1 \alpha_1 + p_2 \alpha_2 = \alpha_1 \{h + (\delta - 1)p_2\}$$

$$J = 2M(k_2^2 \alpha_2 + hH), \quad I = M(k_2^2 + h^2),$$

therefore from (9) we get

$$F(x) \equiv h^3 + (\delta - 1)p_2 h^2 + (2\delta - 1)k_2^2 h - (\delta - 1)p_2 k_2^2 = 0 \dots\dots(i),$$

in which h , p_2 are functions of x .

From $F'(x) \equiv 0$ we have



Fig. 4

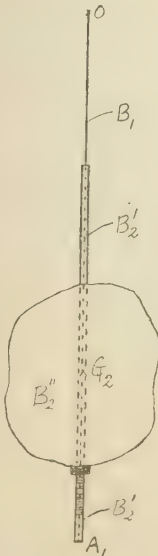


Fig. 5

$$(\delta + 2) h^2 + 2 (\delta - 1) p_2 h + \delta k_2^2 = 0 \dots\dots\dots (ii),$$

and (10) becomes

$$\phi(x, l) \equiv h^2 - l h + k_2^2 = 0 \dots\dots\dots (iii),$$

moreover from $d\phi = 0$ we get

$$(-2h + l) dx - h dl = 0 \dots\dots\dots (iv).$$

Solving (i), (ii), (iii), (iv) simultaneously with respect to h, p_2, k_2^2, dx , we get

$$h = \frac{1}{4} (\sqrt{5} + 1) l \doteq 0.809 l, \quad p_2 = \frac{1}{4(1-\delta)} (2\delta + 1 + \sqrt{5}) l$$

$$k_2^2 = \frac{1}{8} (\sqrt{5} - 1) l^2 \doteq (0.392l)^2, \quad dx = -\frac{\sqrt{5}+3}{4} dl \doteq -1.309 dl, \quad \delta \neq 1$$

and

$$r_1 = h - p_2 = \frac{(\sqrt{5}+3)\delta}{4(\delta-1)} l.$$

Consequently if $\delta > 1$ then $r_1 > 0$, $p_2 < 0$ and the form of pendulum becomes as shown in Fig. 5. And the period of this pendulum is independent of the temperature at any place when the position of B_2'' is suitable.

ON THE SURFACE FIGURE OF THE EARTH, THE MOON AND OTHER PLANETS (Report II)

Saemon Taro NAKAMURA

(Received October 30, 1952)

Chapter II Latitude distribution of land and sea on the earth.

The harmonic expression of the earth's figure given in chapter I¹⁾ approximately agrees to the real distribution of land and sea in low and middle latitudes, but it cannot express the figure in higher latitude or arctic and antarctic regions.

It is because of that the harmonic terms only up to the third order are taken. It must be, therefore, determined at first the highest order of terms to be taken in the harmonic expression to show better figure in polar regions.

To facilitate the calculation, the latitude distribution only are taken in consideration. The latitude distribution of mean h is as shown in Table 1.

Table 1.

North polar distance (θ)	Mean of h (h_0)	North polar distance (θ)	Mean of h (h_0)	North polar distance (θ)	Mean of h (h_0)	North polar distance (θ)	Mean of h (h_0)
0	-1.00	50	0.04	95	-0.33	140	-0.09
5	-1.00	55	-0.07	100	-0.38	145	-0.86
10	0.15	60	0.03	105	-0.42	150	-1.00
15	0.26	65	-0.07	110	-0.36	155	-0.03
20	0.67	70	-0.18	115	-0.35	160	0.18
25	0.76	75	-0.21	120	-0.44	165	0.44
30	0.54	80	-0.31	125	-0.58	170	0.82
35	0.39	85	-0.42	130	-0.75	175	1.00
40	0.31	90	-0.40	135	-0.83	180	1.00
45	0.25						

The harmonic expression of h is as follows

$$h_0 = -0.1125 + 0.1505 \cos \theta + 0.3181 \cos 2\theta - 0.3953 \cos 3\theta - 0.1244 \cos 4\theta - 0.1981 \cos 5\theta \\ + 0.0218 \cos 6\theta - 0.2097 \cos 7\theta - 0.1600 \cos 8\theta - 0.0751 \cos 9\theta. \dots\dots (2)$$

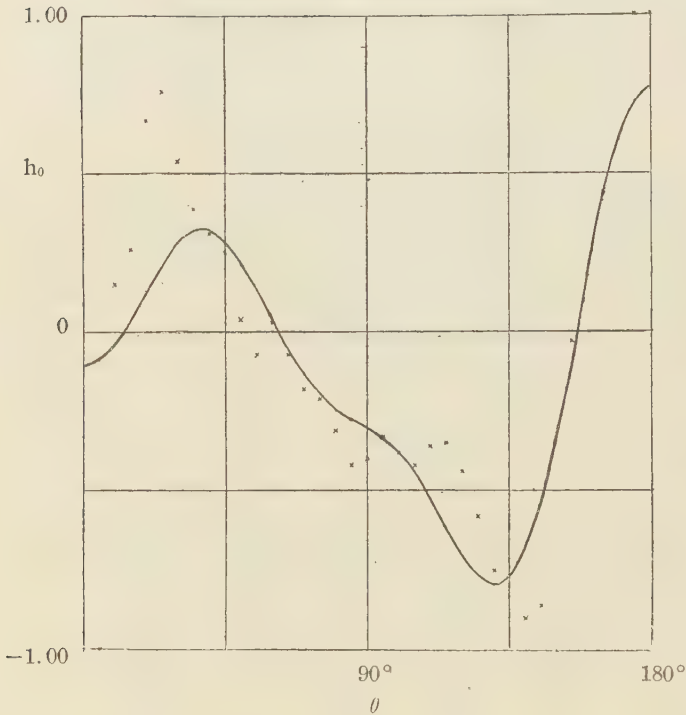
The sum of the squares of errors of this expression, when all terms up to n are taken is as given in Table 2.

Table 2.

n	1	2	3	4	5	6	7	8	9
$\sqrt{\Sigma \varepsilon^2}$	11.64	9.85	6.29	6.09	3.50	3.61	2.54	2.00	1.88

From this table, it may be concluded that the results can be remarkably improved by taking the terms up to 5, but much improvement cannot be expected by taking much higher terms. (see also fig. 2)

Fig. 2.



When h_0 is expressed by

$$h_0 = \sum_{m=0}^5 A_{0m} P_m(\mu) + \sum_{m=1}^5 \sum_{n=1}^m \{ A_{nm} \cos n\varphi P_m^n(\mu) + B_{nm} \sin n\varphi P_m^n(\mu) \}, \dots (3)$$

the coefficients are as shown in Table 3.

Table 3.

$\begin{matrix} n \\ m \end{matrix}$	m	0	1	2	3	4	5
0	A	-0.16951	0.8602	0.0738	0.0888	0.4300	-0.6765
1	A		0.2065	0.3881	-0.0655	0.0743	-0.0068
	B		-0.3197	-0.2654	0.0578	-0.3816	-0.0666
2	A			-0.0487	-0.0971	-0.0042	-0.0093
	B			0.0865	0.1426	-0.0153	-0.0186
3	A				0.0494	0.0085	-0.0008
	B				0.0182	-0.0019	-0.0013
4	A					-0.0017	0.0002
	B					-0.0011	-0.0002
5	A						-0.0004
	B						0.0003

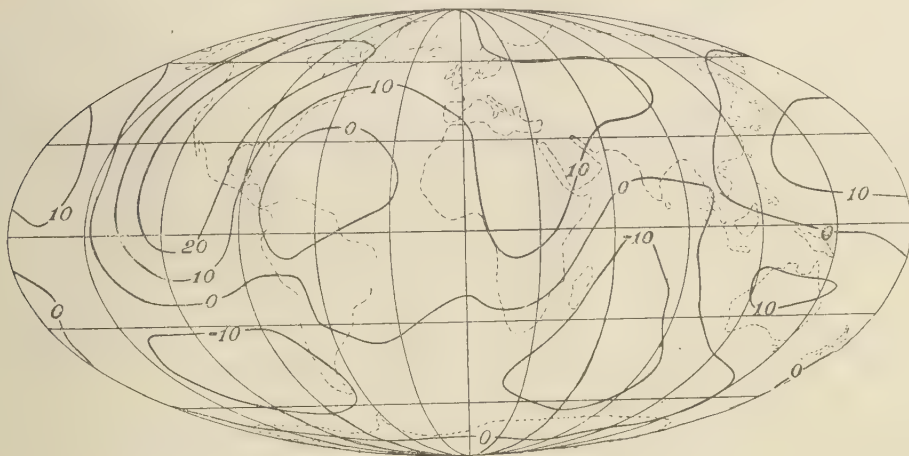
Fig. 3 shows the result computed by using these coefficients. It does not sensibly

Chapter III. Harmonic expression of the surface figure of the earth taking the terms up to the fifth order, using the data of 0-210 E.

As it is too much laborious to calculate all terms up to the fifth order of harmonics at once, as performed in chapter I, the coefficients of the fourth and the fifth orders are calculated by successive approximation. Using the anomalies from equation 1, the terms of the 4th order are calculated. And the terms of the 5th order are then calculated by using the anomalies from thus obtained new equation.

change the conclusion in chapter I that the present figure of the earth itself have some physical significance as a whole as Love's idea.

Fig. 3.



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DISTRIBUTION OF THE MECHANICAL ENERGIES IN THE SOLAR SYSTEM

Saemon Taro NAKAMURA

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The very simple Bode's law of the orbital radii of planets makes us imagine something resembling to the energy distribution in atom model. The author, therefore, computed the mechanical energy of the orbital and of the rotational motions of planets and some large satellites, and found that it is approximately represented by a simple law

$$E = \epsilon^n \dots\dots\dots (1)$$

where E is the mechanical energy and n an integer.

As the present astronomical data available is not sufficient for the calculation of energy, the following assumptions are introduced.

a) The mass of pluto is taken to be 1.5 times of that of the earth. It was estimated from the mass and volume relation in Venus, the earth, uranus and neptune, taking the volume of pluto to be double of the earth.

b) Miner planets are taken as a whole and its mass is assumed to be 1/3000 of the earth. Its radius of orbit is 2.8 astronomical units after Bode's law. The velocity along the orbit is calculated by Kepler's law.

c) As the moment of inertia of planets is not known except for the earth, it is assumed to be expressed by

$$I = CMa^2, \dots\dots\dots (1)$$

as a function of mass (M) and the eqatorial radius (a), and the constant C is assumed to be the same in mercury, venus, the earth, mars and the moon. It is 0,326094. For other planets and for the sun it is assumed to be 0.0771, which is calculated by assuming the density distribution of stars after Eddington.

d) To the rotational energy of planet or the sun, the orbital energy of satellites or that of planets and satellites belongs to it is added.

e) The energy of the proper motion of the solar system as a whole is calculated assuming motion is an obital motion under universal gravity. or it is three times as large as the kinetic energy computed by assuming the velocity of 18.8 km/sec.

f) For the calculation of the orbital energy of planet, mass of planet and satllites belong to it are taken as a whole.

The results of calculation are given in Table 1 and 2.

Table 1 orbital energy

Planets	energy (E)	log E	$n = \frac{\log E}{\log 2}$
Mercury	1.135×10^{40}	40.0550	133.060
Venus	8.989×10^{40}	40.9537	136.052
Earth	8.055×10^{40}	40.9061	135.857
Mars	5.625×10^{39}	39.7561	132.047
Minor planets	9.382×10^{36}	36.9723	122.819
Jupiter	4.868×10^{42}	42.6874	141.804
Saturn	7.932×10^{41}	41.8994	139.187
Uranus	6.044×10^{40}	40.7813	135.473
Neptune	4.586×10^{40}	40.6614	135.074
Pluto	2.963×10^{39}	39.4717	131.122
Solar System	1.063×10^{46}	46.0267	152.897
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Moon	1.136×10^{36}	36.0542	119.770
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" VI	5.639×10^{37}	37.7512	125.407
Neptune I	1.041×10^{38}	38.0175	126.291

Table 2 Rotational Energy

	rotational energy (E_r)	orbital energy of satellites (and planets in sun) ΣE	Total rotational energy (E_1) $= E_r + \Sigma E$	log E_1	$n = \frac{\log E_1}{\log 2}$
Sun	1.426×10^{43}	0.655×10^{43}	2.101×10^{43}	43.3224	143.914
Jupiter	5.750×10^{41}	0.008×10^{41}	5.758×10^{41}	41.7603	138.725
Saturn	1.148×10^{41}	0.001×10^{41}	1.149×10^{41}	41.0603	136.399
Uranus	2.675×10^{39}	0.000×10^{39}	2.675×10^{39}	39.4273	130.975
Neptune	1.675×10^{39}	0.104×10^{39}	1.779×10^{39}	39.2482	130.379
Earth	1.041×10^{37}	0.114×10^{37}	1.155×10^{37}	37.0626	123.021
Mars	3.004×10^{35}	0.000×10^{35}	3.004×10^{35}	35.4771	117.854
Venus	6.307×10^{33}	—	6.307×10^{33}	33.7998	112.280
Mercury	5.393×10^{30}	—	5.393×10^{30}	30.7318	102.089

As seen in the last columns of these tables $\log E_1$ is also nearly the integral power of $\log 2$ or the energy of the orbital as well as the rotational motion is given by

$$E = 2^n \quad (102 < n < 153) \quad \dots \dots \dots (1)$$

where n is an integer. Its probable error is 0.0288.

It is better expressed by

$$E = 0.956 \times 2^n \quad \dots \dots \dots (3)$$

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SOME STUDIES ON VOLCANO ASO AND KUJIU (PART 5) GAS-RUSH IN COAL MINES (PART I).

Munetosi NAMBA.

(Received October 30, 1952)

Abstract.

The phenomenon of gas-rush in coal anthracite mines shows similar characteristics as that of volcanic explosion.

The observation of gas-rush in coal mines enables the writer to reason about certain aspects of volcanic explosion which were not explainable simply by observation of volcanoes.

In this paper he reports mainly about annual and semi-annual variation in gas-rush in coal anthracite mines and volcanoes.

1. Forewords

In coal mines methane gas (CH_4) is always discharged in small degrees. But when energy is a little too powerful, the phenomenon known as 'gas-spout' occurs, and when the energy is more powerful, 'gas-rush' occurs, and then high-compressed methane gas is spouted together with coal blocks.

In the state of gas-rush, some block coals are pushed out with high-compressed methane gas, but in the state of more powerful gas-rush, dust-coal with considerably high temperature are often pushed out. The dust-coal contains a large quantity of methane gas in it. Both gas-rushes are generally accompanied by thunderous sounds.

Immediately before gas-rush occurs, there is an underground rumbling, a roaring sound, and frequently there is a strong vibration of the ground. When gas rush is a powerful one, the stope plane is forced out, and it frequently shows a similar state as the deposit at the upper part of the volcanic pit tube at the time of an eruption. In the case of a volcanic eruption, one cannot observe the state of the inside of the pit tube after the spouting, but in the case of a gas-rush in coal mines, after the dust-coal is spouted, one can clearly observe the fact that the surrounding wall of the cavity has been forced out.

Investigating the above facts, the writer believes that if we draw parallels between the high-compressed methane gas and the volcanic gas that is mainly 'steam', between the coal cave and the volcanic pit tube, and between the dust-coal and the volcanic magma, we may reach some conclusions about the volcanic phenomena which we cannot observe directly with our eyes. Therefore the writer is sure that studies of the gas-rush in coal mines can greatly contribute to volcanology.

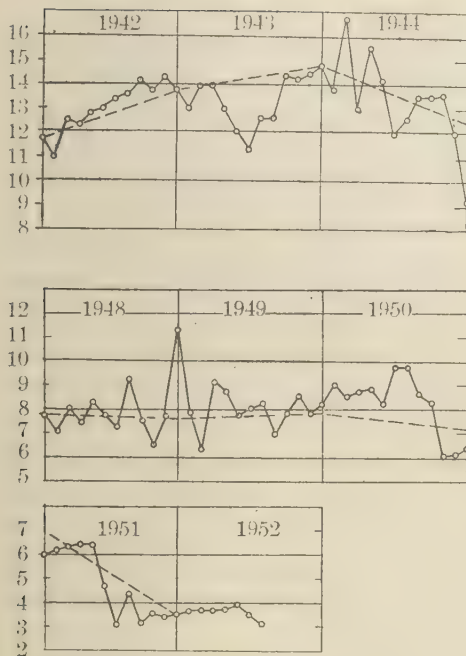
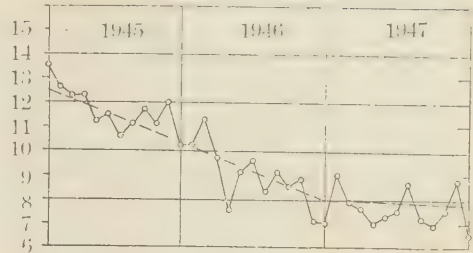


Fig. 1. Gas-discharge per month in the fourth pit (anthracite mine) of Tagawa Coal Mine. unit: -cubic meters per minute.



2. Annual Variations in Gas-rushes.

The gas discharge per month in the fourth pit of Tagawa Mine (anthracite mine) shows the variation curve in annual rate of growth as depict in Figure 1.

This writer can get the same result in studies of records of production of the spouted sulphur at volcano Kujū [1].

Presumably the activity of Volcano Aso shows the same conditions and follows the same processes. Volcano Aso, growing more active in about September, 1932, reached the peak of its recent activity in 1933; since then it has been growing less active in volcanic

gas-rush. It was however impossible to measure the quantity of its volcanic gas-rush, and to get an accurate numerical value of its annual variation of volcanic gas-rush.

Now, this writer found the fact that the action of gas-rush in coal mines was the same as that of volcanic activity, so he is inclined to conclude that the process of volcanic gas-discharge must trace very nearly same process as that of gas-rush in coal mines.

Now, taking up the deviation in the mean curve in the annual rate of growth of methane gas discharge, the writer calculated the annual variation thus :-

$$0.08 \cos (\theta-4) + 0.17 \cos (2\theta-84) + \dots$$

The unit is cubic meters per minute and this result is illustrated in Figure 2. The corresponding annual variation of the atmospheric pressure at Gotōji, the location of the Tagawa Coal Mine, is as follows :-

$$5.63 \cos (\theta-354) + \dots$$

The unit is m.m. Hg.. And then it follows :-

$$0.08/5.6=0.012 \text{ [m}^3\text{/min/m.m. Hg]}$$

Gas-discharge and the atmospheric pressure vary obviously parallel with their annual

4. Diurnal Variation in the Frequency of Gas-rush.

Having studied the conditions of the diurnal variation in the frequency of gas-rushes, this writer found that semi-diurnal variation in the frequency of gas-rushes corresponded with the semi-diurnal variation in the atmospheric pressure as he expected. This result is illustrated in Figure 6.

The result perfectly coincides with the explosions recorded in his paper [9]. Besides it is noteworthy that the frequency of microtremors per hour at the eruption of Volcano Usu-dake (Hokkaidô) as illustrated in Figure 7 represent just the same result as the frequency of gas-rushes per hour do.

Now, the analyses of the frequency of gas-rushes per hour for Tagawa Coal Mine and Shime Coal Mine (Hokkaidô) are respectively as follows :-

$$\begin{aligned}
 &1.7 + 0.84 \cos (t-28) + 1.3 \cos (2t-357) + 1.2 \cos (3t-246) + \dots \\
 &3.6 + 1.3 \cos (t-225) + 1.4 \cos (2t-14) + 2.3 \cos (3t-150) \\
 &+ 1.4 \cos (6t-112) + \dots
 \end{aligned}$$

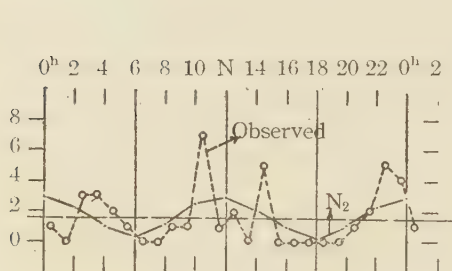


Fig. 6A. Semi-diurnal frequency of gas-rush in Tagawa Coal Mine.

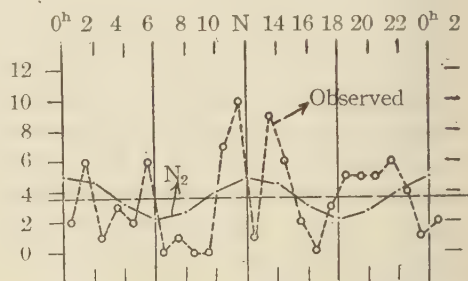


Fig. 6B. Semi-diurnal Frequency of gas-rush in Shime Coal Mine (Hokkaidô).

The term corresponded to the semi-diurnal variation in the atmospheric pressure stands with our expectation. And the other higher terms are expected to vanish in long-dated statistics. In the case of the volcanic activity the reaction of energetic explosion upon the atmospheric pressure variation is just inverse to that of the feeble discharge of volcanic gas. This fact was pointed by this writer in the paper, too. About the same fact in the case of gas-rush in coal mines will report at another time.

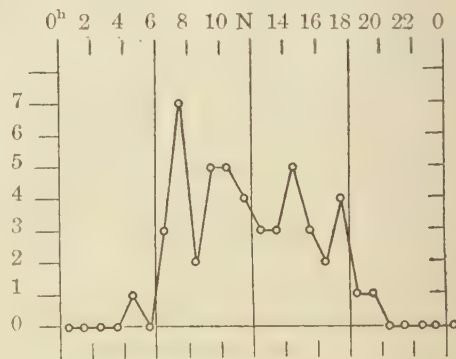


Fig. 7. Frequency of tremors per hour in Volcano Usu-dake (1944 September 1 st. - 3rd.)

5. Conclusion.

1). Gas-rush in coal anthracite mines has all the same characteristics as volcanic explosion.

2). Gas discharge in coal anthracite mines progresses showing a curved line as an annual rate of growth curve. And also volcanic activity is likely to trace the same course.

3). Annual variation of gas discharge occurs parallel with the annual variation in atmospheric pressure. And semi-annual variation of gas-rush occurs slightly after that of the difference of maximum minimum of atmospheric temperature. This seems to this writer to mean that gas discharge is apt to occur at times when atmospheric pressure changes frequently.

4). Studying the numbers of the gas rushes per month this writer learned the fact that powerfull gas-rush changes parallel with the variation of atmospheric pressure like volcanic activity. And the semi-annual variation of gas-rush often occurs at times when the atmospheric pressure changes frequently, diverging from the variation in common rate of the difference of maximum minimum atmospheric temperature.

5). Investigating the frequency of gas-rush per hour in a coal mine, the writer has learned that energetic gas-rush varies parallel with the semi-diurnal variation of atmospheric pressure. This is the same result as Volcano Aso, Kujiu and others.

6). About a weak gas-rush in coal mines the writer will report at another day.

This writer should like to record here his indebttness to professor Sabro Kamzaki for his data about the gas-rush in coal mines. This writer expresses his best thanks to him. This paper was read on October 13 th., 1952 at the annual commitee of Kyushu Branch, Physical Society of Japan.

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Table 1.
Gas-discharge in the Tagawa Coal Mine (Anthracite Pit)

Date	Suply atmosphere in m ³ /min.	Exhaust atmosphere in m ³ /min.	Methane gas in %	Methane gas in m ³ /min.
1942-Jan.	1664	1870	0.63	11.78
Feb.	1667	1829	0.6	10.97
Mar.	1668	1782	0.7	12.47
Apr.	1720	1802	0.68	12.25
May	1728	1829	0.7	12.80
June	1720	1856	0.7	12.99
July	1832	1916	0.71	13.41
Aug.	1736	1889	0.72	13.60
Sept.	1850	1992	0.71	14.15
Oct.	1800	1923	0.72	13.87
Nov.	1872	1976	0.67	14.25
Dec.	1880	1976	0.7	13.87
1943-Jan.	1760	1916	0.68	13.03
Feb.	1744	1923	0.73	14.04
Mar.	1689	1862	0.75	13.97
Apr.	1640	1762	0.74	13.04
May	1372	1608	0.75	12.06
June	1350	1508	0.75	11.31
July	1296	1481	0.85	12.59
Aug.	1280	1407	0.9	12.66
Sept.	1280	1373	1.05	14.42
Oct.	1334	1474	0.97	14.30
Nov.	1398	1527	0.95	14.51
Dec.	1416	1596	0.93	14.84
1944-Jan.	1412	1541	0.9	13.87
Feb.	1564	1681	1.0	16.81
Mar.	1564	1664	0.78	12.93
Apr.	1472	1561	1.0	15.61
May	1482	1574	0.9	14.17
June	1224	1497	0.85	11.96
July	1326	1400	0.9	12.60
Aug.	1322	1498	0.9	13.48
Sept.	1343	1500	0.9	13.50
Oct.	1242	1427	0.95	13.56
Nov.	1193	1329	0.90	11.96
Dec.	1205	1320	0.7	9.24
1945-Jan.	1242	1386	0.98	13.85
Feb.	1117	1341	0.95	12.74
Mar.	1096	1223	1.0	12.23
Apr.	1101	1234	1.0	12.34
May	1120	1247	0.9	11.22
June	1182	1273	0.9	11.50
July	1162	1247	0.85	10.60
Aug.	1117	1223	0.9	11.05
Sept.	1187	1235	0.95	11.73
Oct.	1187	1235	0.9	11.12
Nov.	1188	1260	0.95	11.97
Dec.	1114	1202	0.85	10.22
1946-Jan.	1140	1272	0.8	10.18
Feb.	1155	1297	0.87	11.28
Mar.	1215	1293	0.75	9.74
Apr.	1132	1253	0.6	7.52
May	1225	1297	0.7	9.08
June	1112	1197	0.8	9.58
July	1134	1190	0.7	8.33
Aug.	1156	1197	0.76	9.10
Sept.	1112	1160	0.73	8.47
Oct.	1178	1211	0.73	8.34
Nov.	1161	1291	0.55	7.10
Dec.	1049	1163	0.6	6.97
1947-Jan.	1170	1290	0.7	9.03
Feb.	1195	1320	0.6	7.92
Mar.	1226	1386	0.55	7.62
Apr.	1274	1405	0.5	7.03
May	1371	1468	0.5	7.34
June	1324	1505	0.5	7.53
July	1436	1581	0.55	8.70
Aug.	1350	1443	0.5	7.22
Sept.	1320	1386	0.5	6.93
Oct.	1400	1510	0.5	7.55
Nov.	1475	1760	0.5	8.80
Dec.	1435	1613	0.4	6.45

1948-Jan.	1505	1799	0.45	7.83
Feb.	1490	1767	0.4	7.07
Mar.	1540	1800	0.45	8.10
Apr.	1560	1880	0.4	7.52
May	1620	1864	0.45	8.39
June	1605	1732	0.45	7.79
July	1660	1827	0.4	7.31
Aug.	1710	1850	0.5	9.25
Sept.	1565	1682	0.45	7.57
Oct.	1509	1656	0.4	6.62
Nov.	1737	1949	0.4	7.78
Dec.	1900	2080	0.55	11.44
1949-Jan.	1849	1985	0.4	7.94
Feb.	1980	2130	0.3	6.42
Mar.	1806	2047	0.45	9.21
Apr.	1763	1953	0.45	8.79
May	1797	1960	0.4	7.84
June	1849	2016	0.4	8.06
July	1892	2079	0.4	8.32
Aug.	1672	1744	0.4	6.98
Sept.	1668	1764	0.45	7.94
Oct.	1835	1948	0.45	8.73
Nov.	1863	1970	0.4	7.88
Dec.	2040	2074	0.4	8.30
1950-Jan.	1999	2268	0.4	9.08
Feb.	1956	2142	0.4	8.57
Mar.	1999	2205	0.4	8.82
Apr.	2096	2215	0.4	8.86
May	1935	2079	0.4	8.32
June	2344	2440	0.4	9.76
July	2344	2440	0.4	9.76
Aug.	2021	2170	0.4	8.68
Sept.	1785	1825	0.4	7.30
Oct.	1870	2035	0.3	6.11
Nov.	1892	2072	0.3	6.22
Dec.	1892	2147	0.3	6.44
1951-Jan.	1849	2010	0.3	6.03
Feb.	1892	2077	0.3	6.23
Mar.	1913	2100	0.3	6.30
Apr.	1935	2144	0.3	6.43
May	1935	2146	0.3	6.44
June	1290	1554	0.3	4.66
July	1376	1564	0.2	3.13
Aug.	1376	1480	0.3	4.44
Sept.	1546	1645	0.2	3.23
Oct.	1634	1786	0.2	3.57
Nov.	1634	1780	0.2	3.40
Dec.	1634	1780	0.2	3.40
1952-Jan.	1677	1830	0.2	3.66
Feb.	1720	1830	0.2	3.66
Mar.	1720	1830	0.2	3.66
Apr.	1720	1830	0.2	3.66
May	1828	1928	0.2	3.86
June	1677	1731	0.2	3.46
July	1658	1551	0.2	3.10

Table 2.

A :- Observed mean deviations of gas-discharge (Tagawa Mine, anthracite pit) in cubic meters per minute [7].

B :- Total numbers of volcanic eruptions in Japan [5].

C :- Powerful explosions in the above [5].

D :- Frequency of gas-rush in Tagawa Mine [7].

E :- Frequency of gas-rush in Shime Mine [8].

Month	A	B	C	D	E
1.	0.234	57	6	5	7
2.	0.226	74	6	5	14
3.	0.194	57	7	2	17
4.	-0.042	77	8	7	14
5.	-0.164	53	3	8	7
6.	-0.207	48	1	2	8

Month	A	B	C	D	E
7.	0.026	57	6	2	8
8.	0.043	55	9	2	4
9.	0.087	35	6	3	0
10.	0.028	27	1	3	1
11.	0.001	52	6	3	1
12.	-0.264	42	4	3	3

Table 3. Numbers of gas-rush per hour in Coal mines,

A :- For Tagawa (anthracite pit).

B :- For Shime (No.8 pit).

Hour	A	B
0-1 hour	1	2
1-2	0	6
2-3	3	1
3-4	3	3
4-5	2	2
5-6	1	6
6-7	0	0
7-8	0	1
8-9	1	0
9-10	1	0
10-11	7	7
11-12	1	10

Hour	A	B
12-13	2	1
13-14	0	11
14-15	5	6
15-16	0	2
16-17	0	0
17-18	0	3
18-19	0	5
19-20	0	5
20-21	1	5
21-22	2	6
22-23	5	4
23-24	4	1

THE EFFECT OF TIDAL FORCE ON THE WHITE DWARF STAR

Keisuke KAMINISI

(Received October 31, 1952)

Abstract

As the characteristic equation of the white dwarf, we use the following one:

$$P = A f(x)$$

where $f(x) = x(2x^2 - 3)(x^2 + 1)^{\frac{1}{2}} + 3 \sinh^{-1} x$.

Here the white dwarf can not be a polytrope; nevertheless, it is shown in this paper that a slight tidal effect on the former is similar to that on the latter. Then such a weak tidal force makes almost no change in the degenerate state of the white dwarf.

The characteristics, such as the oblateness of the external shape and the position of the furrow, etc., when $x=1$, come to lie between those of the polytrope with index 1.5 and those with 2.

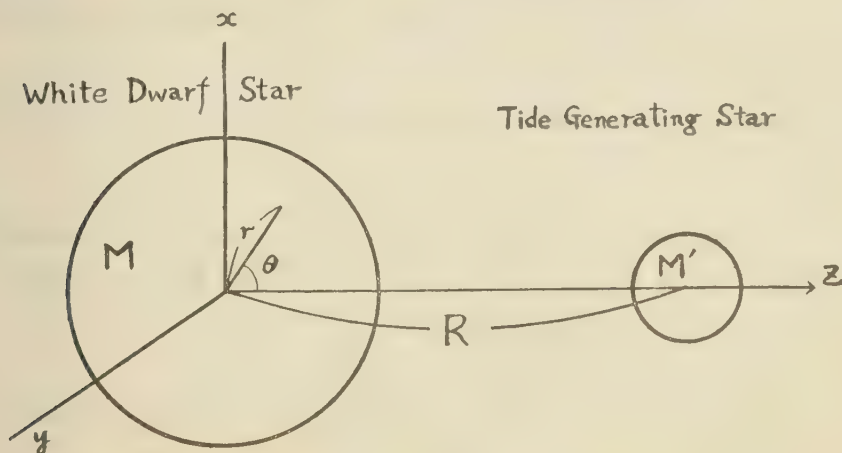


Fig. 1

The above figure illustrates the notations used in this paper.

1. The Equations of the Problem

We use the following characteriatic equation:

$$P = A f(x),$$

where $f(x) = x(2x^2 - 3)(x^2 + 1)^{\frac{1}{3}} + 3 \sinh^{-1} x$,

$$A = \frac{\pi m^4 c^5}{3 h^3}, \quad x = \left(\frac{3}{8\pi}\right)^{\frac{1}{3}} \frac{h}{mc} \left(\frac{\rho}{\mu_e m_H}\right)^{\frac{1}{3}},$$

η_e = Effective molecular weight of electron,

m = Mass of electron,

m_H = Mass of hydrogen atom,

c = Velocity of light,

h = Planck's constant,

ρ = Density.

Hence the fundamental equation of the problem can, by making use of the equations of mechanical equilibrium represented by polar coordinates, be written in the following form [1], [2] :

$$\frac{1}{\eta^2} \frac{\partial}{\partial \eta} \left(\eta^2 \frac{\partial \varphi}{\partial \eta} \right) + \frac{1}{\eta^2} \frac{\partial}{\partial \mu} \left\{ (1 - \mu^2) \frac{\partial \varphi}{\partial \mu} \right\} = - \left(\varphi^2 - \frac{1}{y_c^2} \right)^{\frac{3}{2}}, \quad (1)$$

where $y = \sqrt{1 + x^2}$, $y = y_c \varphi$, $r = \alpha \eta$, $\alpha^2 = \frac{2A}{\pi G B^2 y_c^2}$,

$$B = \frac{8\pi m^3 c^3 \mu_e m_H}{3 h^3}, \quad \mu = \cos \theta, \quad G = \text{Constant of gravitation.}$$

2. Solution

Following the method used by Chandrasekhar [1], we assume the solution of (1) in this form:

$$\varphi = \varphi_0(\eta) + \Phi(\eta, \mu),$$

where $\varphi_0(\eta)$ is the solution of the unperturbed system and $\Phi(\eta, \mu)$ a perturbation term whose second order is negligible. Then φ_0 and Φ satisfy the following equations:

$$\frac{1}{\eta^2} \frac{d}{d\eta} \left(\eta^2 \frac{d\varphi_0}{d\eta} \right) = - \left(\varphi_0^2 - \frac{1}{y_c^2} \right)^{\frac{3}{2}}, \quad (2)$$

$$\frac{1}{\eta^2} \frac{\partial}{\partial \eta} \left(\eta^2 \frac{\partial \Phi}{\partial \eta} \right) + \frac{1}{\eta^2} \frac{\partial}{\partial \mu} \left\{ (1 - \mu^2) \frac{\partial \Phi}{\partial \mu} \right\} = - 3 \left(\varphi_0^2 - \frac{1}{y_c^2} \right)^{\frac{1}{2}} \varphi_0 \Phi. \quad (3)$$

We take for Φ the following form:

$$\Phi = \sum_{j=1}^{\infty} \varphi_j(\eta) A_j P_j(\mu), \quad (4)$$

where A_j is at present an arbitrary constant, and $P_j(\mu)$ is the Legendre polynomial of order j in μ . Substituting (4) in (3) and using the differential equation satisfied by $P_j(\mu)$

and equating the coefficients of the successive orders of Legendre functions, we find that the differential equation defining the radial function $\varphi_j(\eta)$ is

$$\frac{1}{\eta^3} \frac{d}{d\eta} \left(\eta^3 \frac{d\varphi_j}{d\eta} \right) = \left\{ \frac{j(j+1)}{\eta^3} - 3 \left(\varphi_0^2 - \frac{1}{y_c^2} \right)^{\frac{1}{2}} \varphi_0 \right\} \varphi_j \quad (5)$$

To determine the coefficients A 's, we calculate the potential. The result is

$$V = \Gamma \left\{ \varphi - \frac{4}{3} \frac{M'}{M} \eta_s^2 \left| \varphi_s' \right| \left(\frac{\alpha}{R} \right)^{j+1} \eta^j P_j(\mu) \right\},$$

where the suffix s denotes the surface values of the unperturbed white dwarf, and $\Gamma = 4\pi \times GB y_c^3 \alpha^2$. In this calculation, we have assumed that the tide generating star is a mass point i. e., the potential caused by it, is

$$V_{ex} = \Gamma \left\{ \frac{C_0}{\eta} + \sum_{j=1}^{\infty} \frac{C_j}{\eta^{j+1}} P_j(\mu) \right\}.$$

Then by making use of the continuity relations, we find

$$A_j = 0, \quad \text{if } j \neq 2, 3, 4,$$

$$A_j = (2j+1) \eta_s^{j+2} \frac{M'}{M} \left| \varphi_s' \right| \left(\frac{\alpha}{R} \right)^{j+1} \frac{1}{(j+1) \varphi_j(\eta_s) + \eta_s \varphi_j'(\eta_s)}, \quad \text{when } j = 2, 3, 4,$$

Finally we obtain as the solution

$$\varphi = \varphi_0 + \eta_s \left| \varphi_s' \right| \frac{M'}{M} \sum_2^j \nu^{j+1} A_j \frac{\varphi_j(\eta)}{\varphi_j(\eta_s)} P_j(\mu),$$

$$\text{where } \nu = \frac{\alpha \eta_s}{R} \text{ and } A_j = \frac{(2j+1) \varphi_j(\eta_s)}{(j+1) \varphi_j(\eta_s) + \eta_s \varphi_j'(\eta_s)}.$$

3. Degenerate state

The above form of the solution of the white dwarf quite resembles that of the polytrope. We can, therefore, show that both the central density and the mean density of such a star are identical with the unperturbed ones [1], that is, such a weak tidal force can hardly make any change in the degenerate state of the white dwarf, though, of course, there certainly is a considerable change in its shape.

4. Numerical Results : Comparison with the Polytropes [1]

The characteristics of the white dwarf distorted by the tidal force can be completely described by the functions φ_2 , φ_3 and φ_4 .

In this paragraph, however, the numerical integrations of (2) and (5) are carried out only in the one case that $x_c = 1$, where x_c is the value of the parameter x at the center.

We have assumed the above value, for the state of the degenerate gas turns out to be non-relativistic or relativistic corresponding to the value of χ smaller or greater than one.

Numerical values of the functions φ_2 , φ_3 and φ_4 are tabulated in the appendix. In table I we find that the values of J_j and χ in the white dwarf lie between the ones in the polytrope with index 1.5 and the ones with 2. So does the oblateness of the external shape of the white dwarf, which is given by

$$\varepsilon = 1.5 \frac{M'}{M} J_2 \nu^3,$$

lie between the values of the two polytropes corresponding to the index 1.5 and 2.

TABLE I

n	1	1.5	W. D.	2	3	4
J_2	1.51985	1.2500	1.2500	1.1482	1.0289	1.00276
J_3	1.2129	1.1079	1.0904	1.0488	1.00736	1.00047
J_4	1.1205	1.0562	1.0467	1.0231	1.00281	1.00014
χ	0.3995	0.4297	0.4362	0.4567	0.4895	0.4989

n denotes the index of the polytrope and W.D the white dwarf. χ is obtained by

$$\chi = \frac{1}{2} \frac{J_3}{J_2} \nu + 0 (\nu^3)$$

and denotes the angular position (in radian) of furrow on the boundary, which is measured as latitude taking the positive direction of the z-axis (cf. Fig. 1) as the north pole.

It is quite reasonable that we have got the above relations, that is, the characteristics of the white dwarf lie between those of the polytrope with index 1.5 and those with 2, though the white dwarf is not the polytrope, for the polytrope with index 1.5 and that with 3 correspond to the non-relativistic degenerate gas sphere and the completely relativistic respectively and on the other hand the degenerate state of the white dwarf has been assumed as an intermediate state between them.

This paper was read in Sept. 1952 at the regular meeting of Kyushu Branch of the Physical Society of Japan.

Appendix

η	$\varphi_2(\eta)$	$\varphi_3(\eta)$	$\varphi_4(\eta)$
0.0	0.000 000	0.000 00	0.000 000
0.1	0.009 990	0.001 00	0.000 100
0.2	0.039 78	0.007 96	0.001 598
0.3	0.088 84	0.026 71	0.008 069
0.4	0.156 29	0.062 83	0.025 31
0.5	0.240 97	0.121 48	0.061 26
0.6	0.341 44	0.207 33	0.125 74
0.7	0.456 05	0.324 49	0.230 21
0.8	0.583 0	0.476 46	0.387 50
0.9	0.720 4	0.666 1	0.611 5

1.0	0.866 4	0.895 5	0.917 0
1.1	1.018 9	1.166 4	1.319 3
1.2	1.176 1	1.479 7	1.834 0
1.3	1.336 3	1.835 9	2.476 9
1.4	1.497 8	2.235 0	3.263 7
1.5	1.659 1	2.676 6	4.210 1
1.6	1.818 9	3.160 0	5.332
1.7	1.976 1	3.684 4	6.644
1.8	2.129 8	4.248 8	8.162
1.9	2.279 3	4.852 2	9.900
2.0	2.424 1	5.493 6	11.873
2.1	2.563 8	6.172 0	14.097
2.2	2.698 1	6.886 7	16.587
2.3	2.824 5	7.637 1	19.359
2.4	2.948 4	8.423 0	22.429
2.5	3.067 3	9.245	25.815
2.6	3.181 7	10.103	29.536
2.7	3.292 1	10.997	33.611
2.8	3.399 1	11.457	38.063
2.9	3.503 4	12.902	42.918
3.0	3.605 8	13.918	48.203
3.1	3.707 4	14.980	53.950
3.2	3.809 2	16.093	60.194
3.3	3.912 6	17.263	66.982
3.4	4.019 2	18.498	74.367
3.5	4.131 3	19.809	82.420
$\eta_8 = 3.533$	4.169 7	20.262	85.242
$\eta_8 \varphi'_2 (\eta_8) = 4.211 2$		$\eta_8 \varphi'_3 (\eta_8) = 49.023$	$\eta_8 \varphi'_4 (\eta_8) = 306.77$

To obtain the above table, we have carried out the numerical integration of the following equations for $\frac{1}{y_c^2} = 0.5$, by iteration method;

$$\frac{1}{\eta^2} \frac{d}{d\eta} \left(\eta^2 \frac{d\varphi_0}{d\eta} \right) = - \left(\varphi_0^2 - \frac{1}{y_c^2} \right)^{\frac{3}{2}}, \quad (2)$$

$$\frac{1}{\eta^2} \frac{\partial}{\partial \eta} \left(\eta^2 \frac{\partial \Phi}{\partial \eta} \right) + \frac{1}{\eta^2} \frac{\partial}{\partial \mu} \left\{ (1 - \mu^2) \frac{\partial \Phi}{\partial \mu} \right\} = - 3 \left(\varphi_0^2 - \frac{1}{y_c^2} \right)^{\frac{1}{2}} \varphi_0 \Phi, \quad (5)$$

where the boundary conditions have been taken at the center ($\eta=0$) of the white dwarf as follow:

$$\varphi_0 = 1, \quad \frac{d\varphi_0}{d\eta} = 0, \quad \varphi_j = 0 \quad \text{and} \quad \frac{d\varphi_j}{d\eta} = 0.$$

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OXIDATION OF NITRIC OXIDE AND SIMULTANEOUS ABSORPTION BY SULPHURIC ACID

Akira ONUKI* and Ryoichiro TAKAGI**

(Received November 5, 1952)

1. Introduction

Oxidation of NO and simultaneous absorption by H_2SO_4 form the main process of reaction occurring in the Gay-Lussac towers in the production of sulphuric acid. As these towers occupy a huge volume in space, it is most desirable to reduce the total volume by intensifying the reaction and thus to obtain high production capacity.

Up to now, many papers have been written on this subject, but we think none of these is a clear and satisfactory explanation of the phenomena in the towers. Especially concerning the problem of absorption of NO, several research workers have set forth various observations, but it is clear that the rate of the oxidation of NO by O_2 diminishes if the absorption of N_2O_3 by H_2SO_4 occurs simultaneously in the tower. In his study, Leo Berl^[1] has considered this phenomenon only qualitatively, while Kuzuminych^[2] has given the empirical absorption coefficient and Cychikov^[3] has proposed his own equations but with final results which are yet unsatisfactory, mainly due to his neglect of terms in the equation.

The object of this paper is to introduce new equations applicable to this phenomena and to solve these equations verifying its applicability for the present problem. We believe that our new formulae are more practical than those of Kuzminych's, because he had not taken into account the time change of the mol-ratio of oxidation, and we therefore cannot use his formulae for the explanation of whole phenomena.

2. Reaction in the Gay-Lussac tower.

(1) Some properties of nitrogen oxides.

- (i) NO is almost insoluble in H_2SO_4 , but it is absorbed by H_2SO_4 in the form N_2O_3 , when NO_2 coexists.
- (ii) NO_2 dissolves in H_2SO_4 rather well, but it is absorbed by H_2SO_4 almost in the form N_2O_3 when excessive NO coexists.
- (iii) As for the N_2O_3 , equilibrium, $\text{N}_2\text{O}_3 \rightleftharpoons \text{NO} + \text{NO}_2$ holds, but it can be supposed that N_2O_3 is perfectly dissociated at the temperature of towers.
- (iv) As for the N_2O_4 , equilibrium, $\text{N}_2\text{O}_4 \rightleftharpoons 2\text{NO}_2$ holds, but it is permissible to think that the NO_2 is perfectly dissociated as the concentration of the NO_2 is very low and the reaction speed is very high.

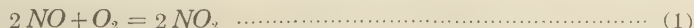
* Physical Institute of Kumamoto University.

**Shin Nippon Chisso Hiryo K. K. Minamata.

In summarizing these properties, we have to take into account only the NO and NO₂ of the nitrogen oxides in the tower, and if we call the state "underoxidation", in which NO's concentration is higher than that of NO₂, in this state the N₂O₃ is absorbed in the H₂SO₄ by the ratio 1 mol of NO and 1 mol of NO₂.

(2) Reaction in the tower.

Gay Lussac towers are packed towers and made up of Kôkaseki* or steel plates, and from the top of them sulphuric acid drops are sprayed down, while gas mixture is blown upwards or downwards through the tubes into it. The main composition of the gas mixture is N₂ but it contains 6.7% O₂ and 2.0% NO+NO₂. We restrict our investigation only to the case of the "underoxidation" state, which is the most usual state of the tower. Then, in order that the absorption of NO by H₂SO₄ may occur, the two following steps are necessary.



Equation (1) was firstly studied by Bodenstein and others [4] [5], according to whom this reaction is a trimolecular one. They have measured the reaction constant for the temperature from 0°C to 290°C.

This is given in Table I.

Table I
Coefficient of Oxidation Velocity

Temp.	30	40	50	60	70	80	90
Kc 10 ⁻⁶	1.59	1.48	1.38	1.31	1.24	1.18	1.12

Later experiment of Hasche and Patrick gives almost the same result, and therefore we think these values are highly credible and have used them in our formulae.

Bodenstein's equation for the rate of oxidation is

$$\frac{dx}{dt} = k_c (a-x) (b-x)^2 \quad \dots\dots\dots (3)**$$

where k_c is the coefficient of oxidation velocity in $\left(\frac{\text{mol}}{l}\right)^{-2} (\text{min})^{-1}$

x is the quantity of O₂ consumed up to time t from time 0 $\left(\frac{\text{mol}}{l}\right)$

a is the concentration of O₂ at $t = 0$

$2b$ is the concentration of NO at $t = 0$

As for the equation (2), keeping in mind only the case of the "underoxidation", we find the equation (2) becomes



*a kind of lava. **Notations used are tabulated in the last page of this paper.

One might consider that the absorption-velocity is governed by the phenomena in the gas film as its speed is very high, so the equation takes the form

$$\frac{dw}{dt} = K_g a (P_g - P_i) \dots\dots\dots (4)$$

Many theories have been propounded about the driving force of absorption. But in our following treatment we assume simply that the driving force is $2 P_{NO}$ when P_{NO} is larger than P_{NO_2} , because Kuzminych [6] and Malin [7] have obtained the same value in their investigations, the former using the statical method, and the latter using the method of wetted wall absorption, and also because our experiment [8] in the packed tower of small size gives nearly the same value.

3. Derivation of the rate equation.

Restricting our problem to the case of underoxidation, we can easily derive the rate equation by using the relation above stated in the following way.

(i) Oxidation term

If we rewrite Bodenstein equation in our notations this becomes

$$\frac{dx}{dt} = - \frac{kc}{120} \left\{ a - \frac{1}{2} (x_o - x) \right\} x^2 \dots\dots\dots (5)$$

But, when the concentration of O_2 is much higher than that of NO , the quantity in this bracket of the right hand side of (5) can be regarded as nearly a constant, and we can assume that (5) takes the form

$$\frac{dx}{dt} = - \frac{kc}{120} a x^2 = - \alpha x^2 \dots\dots\dots (6)$$

where α is constant.

In fact, there is no difference between (5) and (6) under the physical or chemical condition occurring in the towers, and one may practically make use of the equation (6) in place of (5). Secondly, as equation (6) does not contain any relation concerning to absorption, we should add the absorption term to the right hand side of (6). As already remarked above, the driving force of absorption is twice of the partial pressure of NO_2 , so the absorption term becomes

$$\frac{kga}{\rho} (y - r) = \beta (y - r) \dots\dots\dots (7)$$

where β and r are some constants and kga is the coefficient of absorption for $N_2 O_3$ in unit volume, representing the fact that 1 mol of $N_2 O_3$ consists of 1 mol of NO and 1 mol of NO_2 . $2r$ is the partial pressure of $N_2 O_3$ and is assumed to be constant, even though it takes different values from place to place in the tower.

Summing up these terms, the fundamental equations which we must solve should be

$$\begin{aligned}\frac{dx}{dt} &= -\alpha x^2 - \beta(y-r) \\ \frac{dy}{dt} &= \alpha x^2 - \beta(y-r)\end{aligned}\quad \dots\dots\dots (8)$$

$$\text{where } \alpha = \frac{kc}{120} a \quad \beta = \frac{kga}{\rho}$$

4. Solution of the equations

$$\text{Putting } X = \frac{\alpha}{\beta} x \quad Y = \frac{\alpha}{\beta} (y-r) \quad T = -\beta t,$$

the equation (8) takes the dimensionless form

$$\begin{aligned}\frac{dX}{dT} &= Y + X^2 \\ \frac{dY}{dT} &= Y - X^2\end{aligned}\quad \dots\dots\dots (9)$$

and from these it follows

$$\frac{dY}{dX} = \frac{Y - X^2}{Y + X^2} \quad \dots\dots\dots (10)$$

$$T = \int_{X_0}^X \frac{dX}{Y + X^2} = \int_{Y_0}^Y \frac{dY}{Y - X^2} \quad \dots\dots\dots (11)$$

After integrating the equation (10), one can easily get X and Y as the function of T from (11), but unfortunately it can easily be proved that (10) cannot be solved by any elementary function. (One can verify that eq. (10) is transformed to Riccati type equation). Therefore, we must solve (10) by numerical method under suitable boundary conditions. In order to do this, it is necessary to determine the values of the coefficient in the equation first, and then assume initial values of variables in the expression. Examples of these procedures and results of calculation are given in the following section. We have used Runge-Kutta's integration method for the numerical treatment, but in some cases the iteration method is more available and is recommended.

5. Example of Calculation.

In order to determine the constants α , β , and γ , we take as the conditions; Temp. 40°C and 1 atm. pressure and the composition of inlet gas to 2nd tower is

NO+NO ₂	0.6375%
NO	0.439%
NO ₂	0.1985%
O ₂	6.5%

and the value of absorption coefficient is equal to $3 \frac{\text{Kg mol}}{\text{m}^3 \text{hr atom}}$ and the vapour pressure of the N_2O_3 on nitrose is 0.112 mm Hg.

From these values we get α , β , γ as follows

$$\alpha = 31.2 \frac{(\text{mol}^{-1})}{l} (\text{sec})^{-1}$$

$$\beta = 0.0305 (\text{sec})^{-1}$$

$$\gamma = 3.30 \cdot 10^{-6} \left(\frac{\text{mol}}{l} \right)$$

Calculations have been made using these constants and some initial values of variables, the results of which are given in Fig. 1 and 2. To compare these results with other case, three curves for different values of β are given in the same Figure, and our numerical example corresponds the curve marked by (No. 3).

6. Discussion of results.

Under the condition stated in the previous section, experimental total loss of HNO_3 which we have obtained in the factory was 8.5kg/ton of 100% H_2SO_4 for practical production. On the one hand our calculation shows this loss of HNO_3 with tail gas must be 7.6kg/ton which one can consider rather good agreement of theory and experiment, taking into account other miscellaneous losses of HNO_3 in the procedure. In addition to this, searching for the composition of tail gas from 5th tower after 200 seconds from the beginning, we get the following values easily from Fig. 1 and 2.

	Kga	NO	NO ₂	NO+NO ₂	Ω
No. 1	12	0.121	0.011	0.132	8.5%
2	6	0.102	0.011	0.113	9.5%
3	3	0.082	0.012	0.094	12.7%
4	1.2	0.065	0.048	0.113	42.5%

From these values it can be concluded that there will be one suitable value of β which makes the concentration of NO+NO₂ in the tail gas minimum, i. e. a suitable value of Kga at which total loss of HNO_3 becomes minimum. In our example of the section 5 above, the suitable value of Kga is 3 which also shows good agreement with our experimental data in the factory. Nevertheless, one should not forget that this circumstance is only realizable under the assumption that 4th and 5th tower have the same Kga, and consequently, it can be supposed that a more advantageous way is to make the 4th tower's Kga smaller in some degree and accelerate the oxidation, and on the contrary to make Kga larger in the 5th tower.

Moreover, our calculation shows that the whole absorption process is mainly governed

by the oxidation velocity in case Kga is not too small, and that the large part of the loss of HNO_3 comes from unabsorbed NO . The measured value of Kga for Gay-Lussac tower is from 1. to 3. in $\frac{\text{Kg mol}}{\text{m}^3 \text{ hr. atm.}}$.

7. Influences of variation of the condition.

i) Oxidation Velocity. By lowering the temperature the constant kc becomes large and its increasing rate is about 10% per 10°C . As the change of the oxidation velocity is proportional to the concentration of O_2 and also to the square of concentration of NO , it is generally preferable to increase the former one, but in many cases it is rather important to keep the initial concentration of NO constant, because a small change in the latter causes remarkable variation of oxidation velocity. If the concentration of NO changes appreciably, it becomes almost impossible to hold the most suitable condition for the absorption throughout the process in the tower and the loss of HNO_3 increases in consequence of it.

ii) Absorption Velocity.

When the constituents of gas at the entrance and the reaction time in the towers are known, the most suitable value of Kga will be determined from our calculation. It is often preferable to lower the value of Kga for such towers. The ratio α/β determines the relation

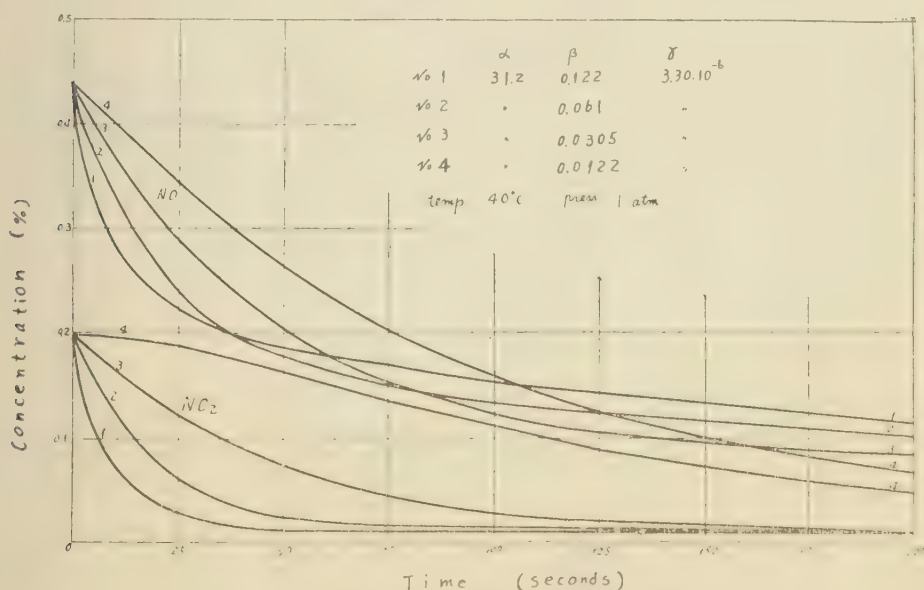


Fig. 1 Concentration Curves of NO and NO_2

of x to y , and consequently the shape of curve, assuming the initial value of x_0, y_0 . If α is constant, one will get easily from the figures (Fig. 2) the value of β which makes the concentration of $\text{NO} + \text{NO}_2$ minimum at the outlet.

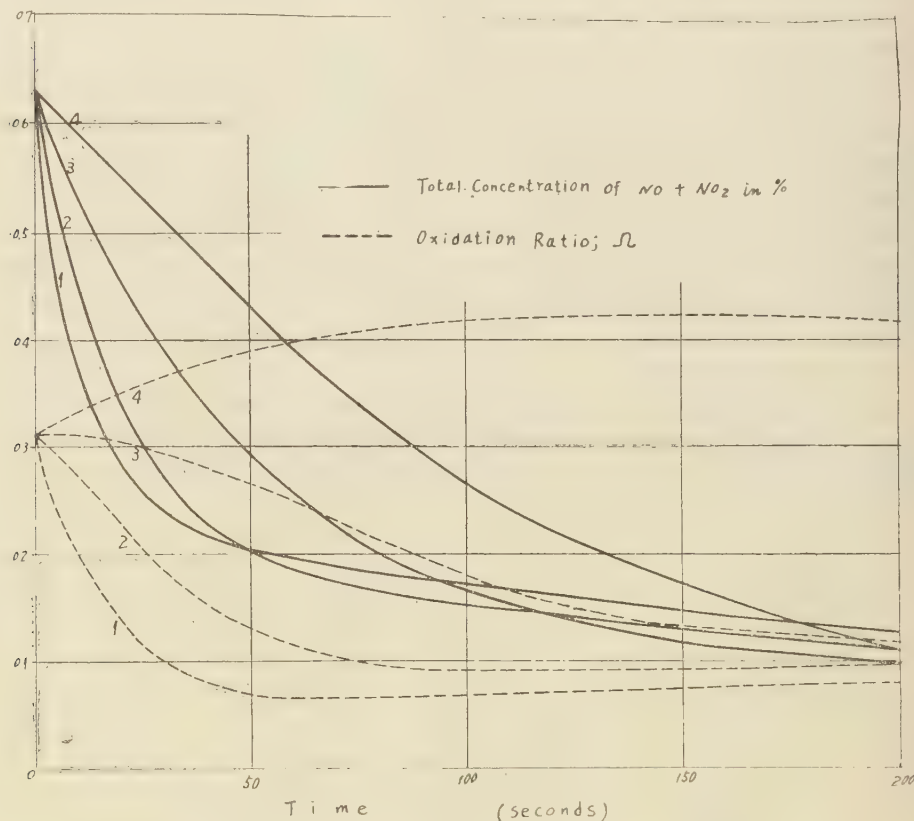


Fig. 2 Total Concentration and Oxidation Ratio Curves

8. Conclusions and Summary

We have proposed the new equations for the reaction occurring in the Gay-Lussac towers. These are

$$\begin{aligned}\frac{dx}{dt} &= -\alpha x^2 - \beta(y-r) \\ \frac{dy}{dt} &= \alpha x^2 - \beta(y-r)\end{aligned}$$

Numerical methods were used to solve the equations verifying its applicability for the calc-

ulation of the loss of HNO_3 in the procedure. Results show good agreement with the experimental data.

Discussion has been provided in some detail. We hope our new formula will contribute to some improvement of the planning and operation in the sulphuric acid manufacturing plants.

We wish to express our thanks to Mr. Takuro Kamiya and Mr. Tsuneo Kato of the Shin Nippon Chisso Hiryo K. K. Minamata, who gave help in many respects.

NOTATIONS

x	: Concentration of NO in the gas	mol/l
y	: " of NO_2 "	"
x_0	: Initial concentration of NO in the gas	"
y_0	: " " of NO_2 "	"
a	: " " of O_2 "	"
t	: time	sec
α	: Coefficient of oxidation term $(\text{mol/l})^{-1}(\text{sec})^{-1}$	
β	: " of absorption " $(\text{sec})^{-1}$	
2γ	: the partial pressure of N_2O_3 on nitrose	mol/l
kg_a	: overall absorption coefficient	sec^{-1}
Kg_a	: overall absorption coefficient	$\text{kg mol/m}^3 \text{ hr atm.}$
ρ	: Meanratio of hollow space in a tower.	
Ω	: Oxidation ratio, namely the ratio of the concentration of NO_2 to the total concentration of $\text{NO} + \text{NO}_2$.	
kc	: Coefficient of the oxidation velocity at NO $(\text{mol/l})^{-2} (\text{min})^{-1}$	
dw/dt	: Absorption velocity of N_2O_3	(mol/h) .
P_g	: Partial pressure of N_2O_3 in the gas	(atm.) .
P_i	: Vapour pressure of N_2O_3 on nitrose	(atm.)

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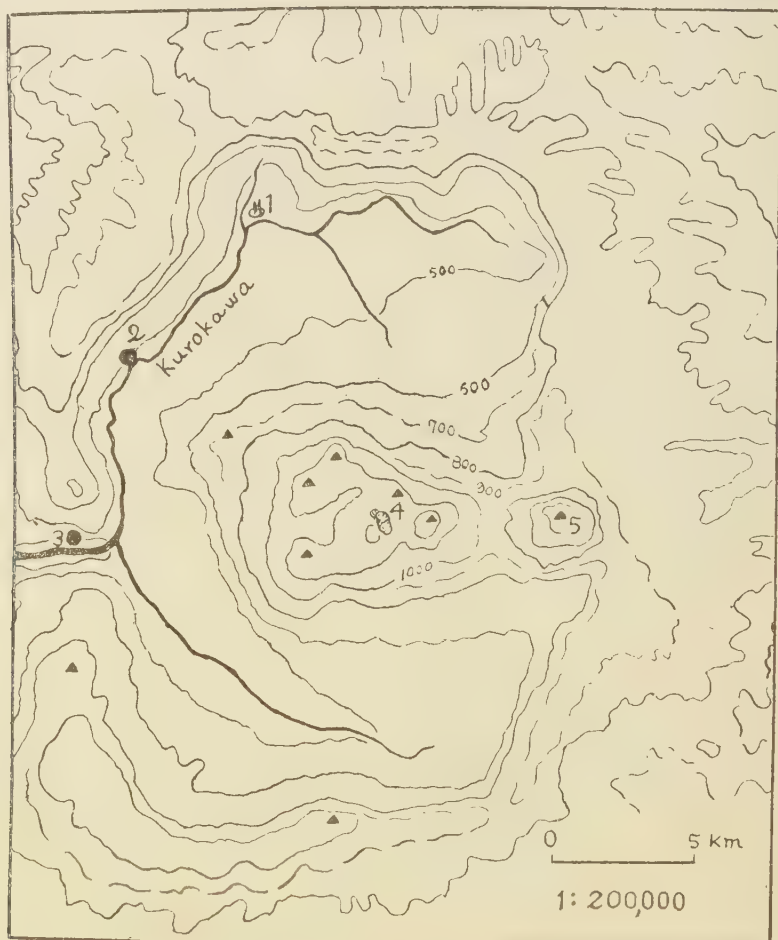
SOME STUDIES ON VOLCANO ASO AND KUJIU (PART 4) ON THE VRAIATION OF THE WATER HEAD AT KUROKAWA, ASO

Tosisato MUROTA

(Received October 10, 1952)

Abstract

This is a study about the real aspects and the cause of the diurnal variation of the



1. Uchinomaki, 2. Matoishi, 3. Tateno, 4. Nakadake, 5. Nekodake,

Fig. 1. Aso Crater Basin

water head on the no-rainy days, comparing and examining the records of the water head at the two spots: Uchinomaki and Matoishi which both stand by the River Kurokawa which is running inside the Aso crater basin.

1. Foreword

Even though there are many reports on study about the relation between precipitation and quantity of flow, the reports about the variation of the river water head on the no-rainy days are very scarce, and more than that they are about the variation of the river water head caused by increase and decrease of the amount of the snow-water [1]. About the diurnal variation of the river water head throughout a year, Dr. Nomitsu indicated that there is a regular diurnal variation on the water head on the no-rainy days also, and he reported about the real aspects and the cause of the diurnal variation of the water head at the River Kurokawa, using the records of the water head throughout a year from June 1940 to May 1941 at the Matoishi water gauge station at Kurokawa, Aso [2].

The writer investigated (1) annual change in the diurnal variation of the water head, (2) semi-annual change in the diurnal variation of the water head, (3) variation of time of appearance of the highest water head, and (4) relation between the spot of observation and the variation of the water head, by comparing and studying the records of the water head throughout a year from July 1948 to June 1949 at the Uchinomaki water gauge station which stand by the same Kurokawa and the records at the Matoishi station which Dr. Nomitsu had used.

2. Analysis of the Observed Data

The water head was recorded on an endless revolving horizontal cylinder in natural scale. Observed values are as given in Table 4 at the end, and Fig.2 shows some examples of the records.

Using the records in the days which do not show the hasty change by the rainfall, diurnal variations of the water head are calculated, taking the influence of the natural decrease of water head into consideration and making the water head at midnight zero. and the results are presented in Table 3 at the end. The results of harmonic analysis of the monthly mean at each o'clock in Table 3 are given in Table 1.

The following equations show the results of harmonic analysis of the diurnal variation in Table 1 with respect to the month (t shows month).

Diurnal variation of the water head at Uchinomaki is:

A_1 (amplitude)

$$0.54 + 0.13 \cos(t - 174.7) + 0.30 \cos(2t - 77.3) + 0.12 \cos(3t - 134.5) + \dots$$

θ_1 (phase)

$$63^\circ.2 + 55.7 \cos(t - 72.9) + 80.1 \cos(2t - 319.9) + 49.8 \cos(3t - 345.9) + \dots$$

Table 1.

Uchinomaki								Matoishi							
Month	A ₀ (cm)	A ₁ (cm)	θ ₁ (deg)	A ₂ (cm)	θ ₂ (deg)	A ₃ (cm)	θ ₃ (deg)	A ₀ (cm)	A ₁ (cm)	θ ₁ (deg)	A ₂ (cm)	θ ₂ (deg)	A ₃ (cm)	θ ₃ (deg)	
1	0.622	0.178	171.3	0.165	191.7	0.225	303.1	-0.403	0.514	352.0	0.218	265.8	0.009	171.6	
2	-0.708	1.004	64.2	0.309	162.1	0.436	22.3	-0.894	0.918	14.0	0.288	258.9	0.064	81.2	
3	-0.582	0.824	23.8	0.075	117.1	0.163	110.1	-0.366	0.230	348.7	0.089	268.7	0.071	6.9	
4	-0.347	0.145	35.6	0.133	45.3	0.285	90.2	0.553	0.552	243.8	0.397	193.5	0.014	15.7	
5	0.588	0.483	193.2	0.360	122.1	0.013	123.8	1.130	1.313	217.6	0.215	264.1	0.095	197.1	
6	0.205	0.508	108.5	0.031	161.2	0.179	121.1	0.866	0.947	201.0	0.045	318.1	0.032	254.3	
7	-0.349	0.616	88.6	0.232	330.0	0.414	289.6	1.195	1.408	147.9	0.203	267.9	0.085	301.7	
8	-1.100	1.031	5.8	0.023	335.6	0.147	295.2	1.442	1.940	147.4	0.114	301.2	0.058	308.8	
9	-0.717	0.764	327.4	0.035	202.5	0.037	292.7	0.188	1.098	116.4	0.250	33.1	0.128	105.7	
10	-0.397	0.298	343.1	0.098	106.7	0.058	359.4	0.349	0.507	138.4	0.050	296.6	0.069	67.2	
11	0.497	0.498	289.6	0.255	203.2	0.054	30.2	0.294	0.206	156.2	0.058	156.0	0.034	144.9	
12	0.009	0.112	187.5	0.229	125.8	0.202	255.8	-0.008	0.185	283.9	0.080	263.5	0.062	162.1	

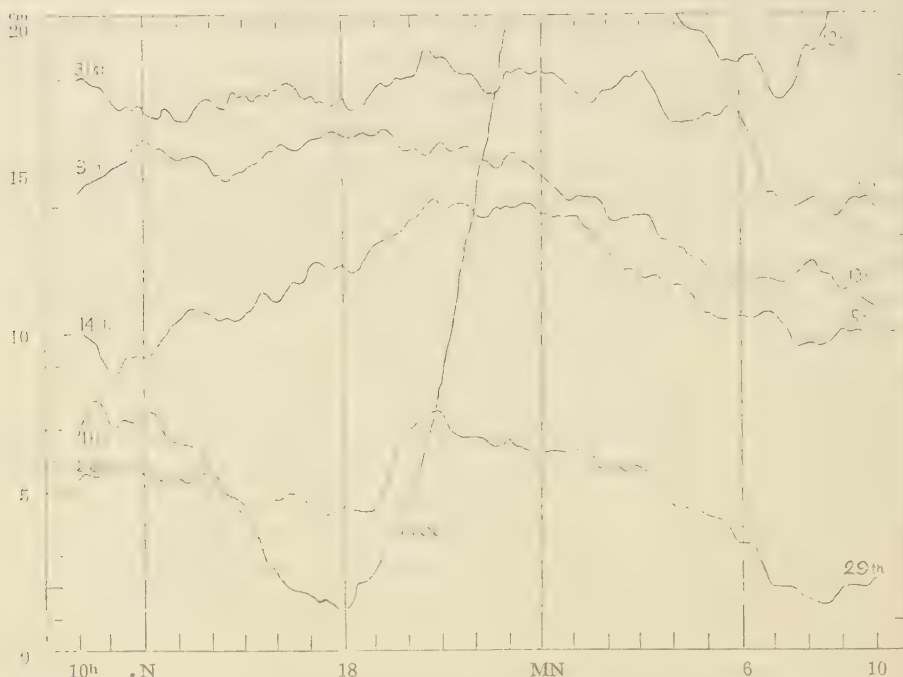


Fig. 2. Some Examples of the Records of Water Gauge
(From Aug. 31 to Sep. 29, 1948)

Diurnal variation of the water head at Matoishi is:

A₁ (amplitude)

$$0.82 + 0.57 \cos(t - 178.0) + 0.33 \cos(2t - 42.9) + 0.19 \cos(3t - 343.2) + \dots$$

θ₁ (phase)

$$227.3 + 117.5 \cos(t - 37.7) + 38.4 \cos(2t - 28.8) + 7.3 \cos(3t - 49.3) + \dots$$

Fig. 3 and 4 show these in diagram.

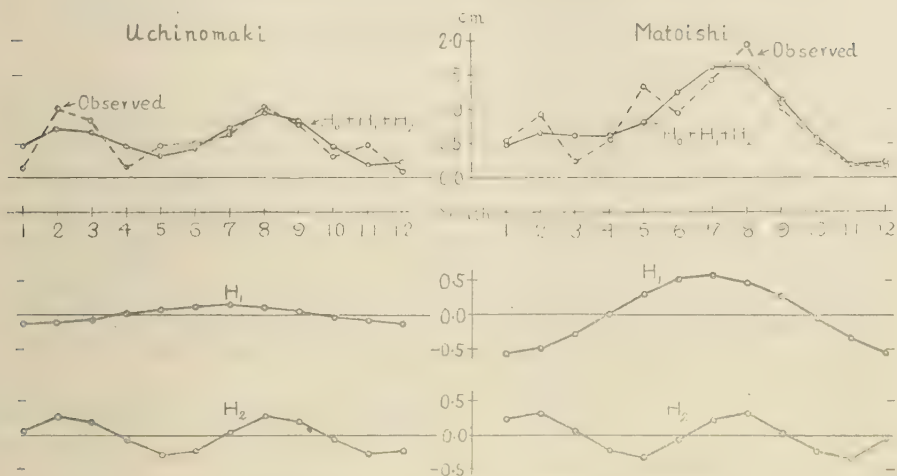


Fig. 3. Amplitude of the Diurnal Water Head Variation

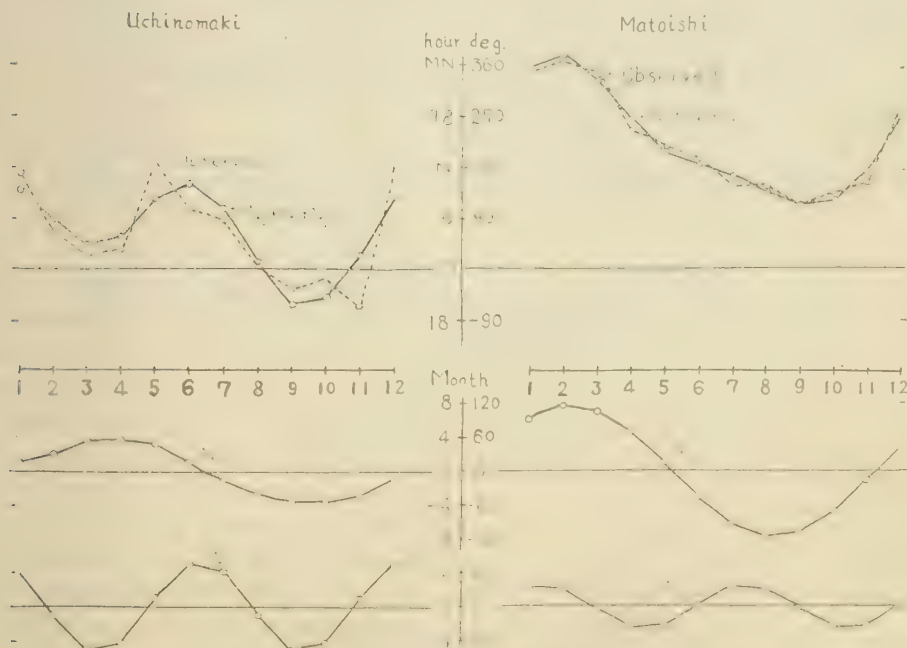


Fig. 4. Time of Appearance of the Highest Water Head

3. Climatic Elements Observed at Aso and their Analysis

The monthly mean of climatic data during ten years from 1930 to 1939 at Aso is shown in Table 2 [3].

Table 2.

Month	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Atm. Temp. °C	1.2	2.3	5.7	11.1	15.6	18.9	22.8	23.2	19.2	14.1	9.2	5.8
Evap. Amount mm	1.7	2.0	3.1	3.7	4.4	3.8	4.0	4.5	3.4	3.0	2.4	1.8
Vap. Tension mm	3.8	3.9	4.9	7.2	9.3	12.6	17.1	16.7	13.1	8.9	6.5	4.7
Rel. Humidity %	74	73	70	71	71	78	84	78	79	73	72	71
Atm. Press. +700mm	16.7	14.9	14.2	13.5	12.4	10.1	10.8	10.8	12.3	15.0	16.7	16.9

For to see its annual and semi-annual variation, we express them in the following expressions.

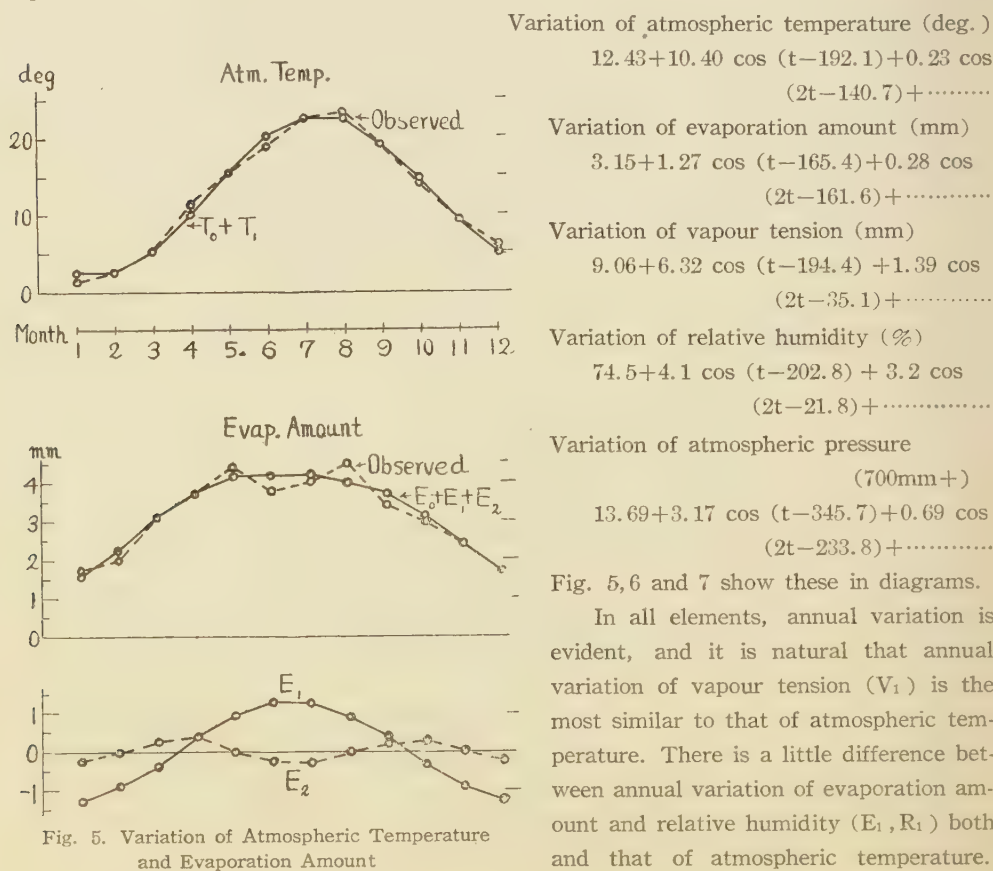


Fig. 5. Variation of Atmospheric Temperature and Evaporation Amount

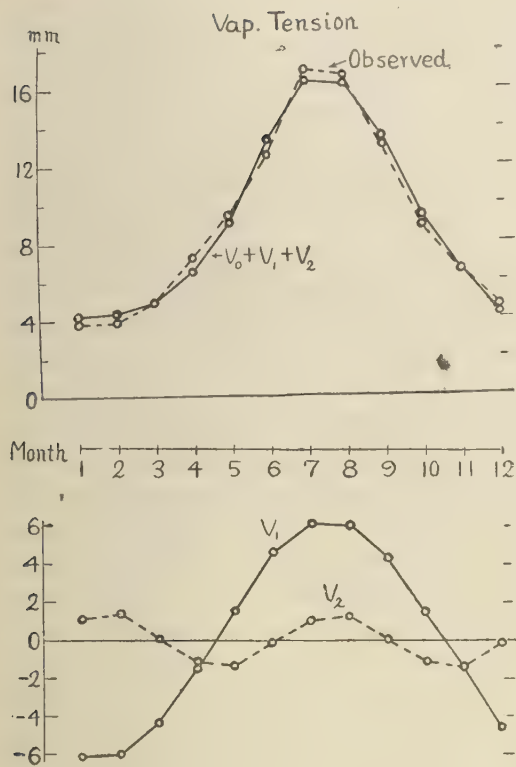


Fig. 6. Variation of Vapour Tension

This shows that there is some influence of something besides atmospheric temperature on evaporation amount and relative humidity.

On other elements besides atmospheric temperature, semi-annual variation is recognized clearly and only atmospheric temperature does not show semi-annual variation, so from this it is considered that semi-annual variation which is recognized in other elements is caused by something which is different from atmospheric temperature. Also, among these, semi-annual variations of vapour tension and relative humidity (V_2, R_2) are different from others with their specially similar phases. This variation shows that its maximum is seen about February and August and minimum about May and November. We notify this specially for the discussion following.

4. Diurnal Variation of the Water Head

Dr. Nomitsu already indicated that diurnal variation of the water head in warm season at Matoishi is mainly caused by the action of transpiration of plants about the region of the source of the river and diurnal variation of the water head in cold season is mainly influenced by the thawing of snow [4]. We got the following results from comparing and studying of data at Uchinomaki which stands about the middle point along the River Kurokawa and Matoishi which stands about ten kilometers below from it.

(1) Comparison of yearly mean of amplitude of diurnal variation of the water head is :-

$$\frac{0.54 \text{ (Uchinomaki)}}{0.82 \text{ (Matoishi)}} = 0.66$$

Comparison of area of basin is :-

$$\frac{95\text{km}^2 \text{ (Uchinomaki)}}{167\text{km}^2 \text{ (Matoishi)}} = 0.57$$

From this, it is considered that yearly mean of amplitude of diurnal variation of the water

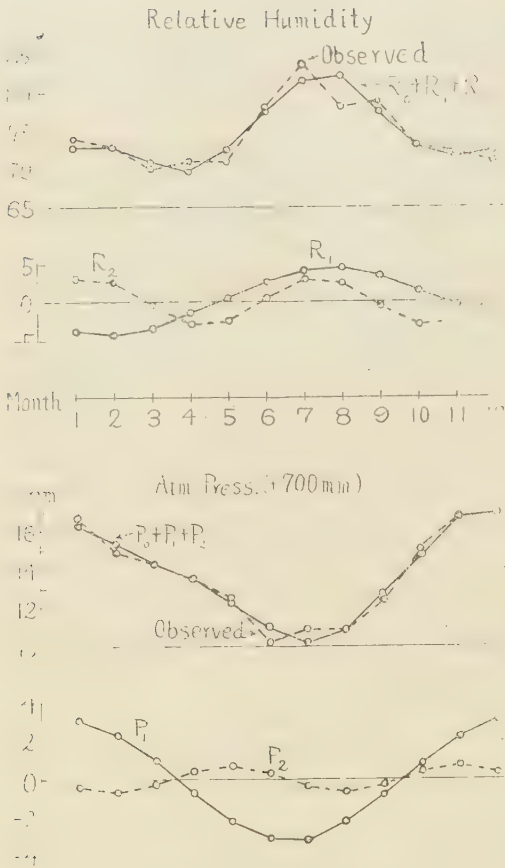


Fig. 7. Variation of Relative Humidity and Atmospheric Pressure

head is almost in proportion to extent of area of basin.

(2) An observation of diagram H_1 in Fig. 3 shows that the top of annual change rises in July both at Uchinomaki and Matoishi. Comparing Fig. 3 with Fig. 5 and 6, it is clear that annual change in diurnal variation of the water head has relation, to annual change in atmospheric temperature and vapour tension. And also it is understood that its relation means that action of transpiration of plants and evaporation from the ground change themselves in response to change in atmospheric temperature, and give influence to the water head. About amplitude of annual change in diurnal variation of the water head, it is 0.13 cm at Uchinomaki and 0.57 cm at Matoishi, and so Matoishi has more value of about four times than Uchinomaki. This shows that at Matoishi diurnal variation of the water head is large in warm season and small in cold season, comparing with Uchinomaki. The cause of this difference on diurnal variation, even though they change in the similar phases under the same causes, may be ascribed to the difference

of the conditions of basins around the observation points. Namely, the difference results from the following. There are comparatively rather many latifoliate trees in the area around Matoishi, wherefore transpiration in the daytime in summer season increases specially and that makes diurnal variation of the water head larger. While on the contrary, there are comparatively rather many acerose trees in the area around Uchinomaki, wherefore transpiration in the daytime in summer season is not so strong as Matoishi and diurnal variation is smaller than at Matoishi, but in cold season transpiration is strong comparing with Matoishi and so diurnal variation at Uchinomaki is larger than at Matoishi. Eventually annual change in diurnal variation of the water head at both points, is influenced indirectly by change in atmospheric temperature, but there is difference in condition which intermidi-

ates this influence and this gives a large difference in the amplitude of diurnal variation of the water head according to the season even though phases of annual change in the diurnal variation of the water head are same.

(3) Semi-annual change in diurnal variation of the water head (H_2) has the similar amplitude and phase at Uchinomaki and Matoishi, as it is shown in Fig. 3. This results from the close resemblance of condition of the two areas with respect to the influence of annual plants (especially cereal glasses) and there is no relation with location of the observation points. Also the resemblance between the diagram H_2 in Fig. 3 and the diagram V_2 in Fig. 6 (semi-annual change in vapour tension) shows the cause of semi-annual change H_2 is same with the cause of semi-annual change in vapour tension. An examination in the diagram H_2 and V_2 shows that the maximum is seen in February and August and the minimum in May and November, and it was already indicated in article 3. The fact just described may be explained as follows. The minimum in May is caused by lack of underground water as the plant's growth increases (specially the annual plants) in that season. The maximum in August is attributed to surplus of water which results from decrease of absorptive power of water as the plants grow the most then and also from strong evaporation and transpiration during the daytime. The minimum in November shows lack of moving water in the ground in the season begining to freeze. And the maximum in February results from that surplus of water melted from ice in the daytime gives influence to diurnal variation of the water head.

Amount of transpiration is usually, about 100–230 mm a year and for example it is as following: - acerose tree 100 mm, shrub 150 mm, latifoliate tree 200 mm and cereal plant 230 mm (5). The difference of amount of transpiration between acerose tree and latifoliate tree, fact that transpiration of latifoliate tree is vigour in the warm season, and fact that transpiration of cereal plant is almost limited in the warm season are the powerful supports of the discussion previously mentioned.

(4) Phase of diurnal variation of the water head (time of appearance of the highest water head) and result of its analysis are shown in Fig. 4. Difference of yearly mean of phase is :-

$$227^{\circ} \text{ (Matoishi)} - 63^{\circ} \text{ (Uchinomaki)} = 164^{\circ} \rightarrow 11 \text{ hours.}$$

On an average, time of appearance of the highest water head at Matoishi at the lower reaches of the river is about 11 hours later than at Uchinomaki at the middle of the river. This can be understood, considering that distance between Uchinomaki and Matoishi is about 10 km, together with the speed of current.

In Fig. 4 annual change and semi-annual change (N_1, N_2) are recognized distinctly and both show similar tendency at Uchinomaki and Matoishi.

About annual change (N_1), phase advances from spring to summer and lags from autumn to winter. This depends on that, phase becomes opposite in warm season and cold season

because action of absorption of water by transpiration in warm season is just opposite to action of snow-water in cold season.

About semi-annual change (N_2), phase advances sometime and lags sometime as following.

Winter $\xrightarrow{\text{advances}}$ spring $\xrightarrow{\text{lags}}$ summer $\xrightarrow{\text{advances}}$ autumn $\xrightarrow{\text{lags}}$ winter

In comparing this change with time of the sunrise and sunset shown in Fig. 8, the following facts are recognized :- phase changes according to the change of time of the sunrise in

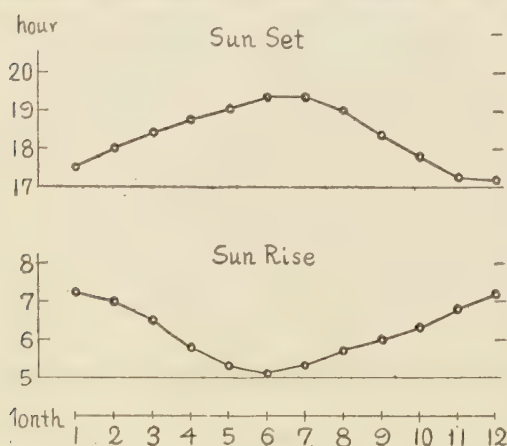


Fig. 8. The Time of Sunrise and Sunset on the 15th in Every Month at Kumamoto

cold season in which action of thawing is the main cause of diurnal variation of the water head, and phase changes according to the change of time of the sunset in warm season in which action of transpiration of plant and evaporation from ground is the main cause of diurnal variation of the water head. In this reason, this matches with the conclusion that the main cause of diurnal variation of the water head in warm season is transpiration and evaporation and the main cause of diurnal variation of the water head in cold season is thawing.

5. Conclusions

The following are the main points of the statements above.

(1) Annual mean of amplitude of diurnal variation of the water head at Uchinomaki which stands at the middle bank of the Kurokawa River, Aso and Matoishi which stands at its lower stream is almost in proportion to extent of area of basin.

(2) Annual change in diurnal variation of the water head results from indirect influence of annual change in atmospheric temperature, and magnitude of the amplitude of diurnal variation is influenced by the kinds of trees in forest near basins around the observation points.

(3) Semi-annual change in diurnal variation of the water head almost does not have relation with situation of the place of observation, but influenced mainly by action of transpiration of the annual plants like cereal glasses in warm season and by freezing and thawing of water in ground in cold season.

(4) Time of appearance of the highest water head, on average, lags at the lower

stream. Phase of its annual change becomes opposite in warm season and cold season, and it is recognized that semi-annual change changes according to change of time of the sunset in warm season and change of time of the sunrise in cold season.

In concluding this paper the writer wishes to express his hearty thanks to the following persons : records of water gauge were made by efforts of H. Takehara, M. Watana be, T. Eto and S. Mori during ten years; on adjustment T. Eto, M. Watanabe and T. Narakino helped it, and Dr. M. Namba gave kind advices to the writer.

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Table 8 (1)
Reduced Diurnal Variation of the Water Head at Uchinomaki (cm)
(from July 1948 to June 1949)

Jan.

[illegible]

Feb.

[illegible]

Day	9	17	19	20	22	23	24	25	26	28	29	30	31	Total	Mean
1	0.28	0.45	0.37	0.95	0.11	0.61	0.46	0.08	0.42	0.22	0.09	0.34	0.45	0.00	0.098
2	0.24	0.50	0.27	0.91	0.11	0.30	0.68	0.15	0.27	0.21	0.59	0.46	0.58	0.3	0.069
3	0.61	0.25	0.63	0.47	0.22	0.79	1.00	0.69	0.36	0.24	0.78	0.77	0.77	0.6	0.481
4	0.69	0.25	0.17	0.20	0.84	0.20	1.50	1.50	0.38	0.15	0.03	0.05	0.33	10.638	0.519
5	0.59	0.32	0.97	0.25	0.59	0.30	1.51	0.63	0.43	0.35	0.35	0.49	0.69	4.438	0.579
6	0.35	0.18	0.31	0.25	0.25	0.30	0.95	0.14	0.33	0.17	0.00	0.00	0.04	2.637	0.275
7	0.41	0.18	0.72	0.69	0.21	0.53	0.50	0.53	0.42	0.38	0.21	0.36	0.52	2.637	0.275
8	0.67	0.10	0.70	0.69	0.30	0.33	0.52	0.73	0.33	0.43	0.00	0.00	0.00	2.637	0.275
9	0.33	0.40	0.88	0.95	0.30	0.34	0.07	1.21	0.20	0.40	0.00	0.00	0.00	2.637	0.275
10	0.05	0.05	0.33	0.95	0.05	0.04	0.10	0.62	0.52	0.19	0.00	0.00	0.03	2.637	0.275
11	0.53	0.15	0.47	0.05	0.44	0.56	0.50	0.00	1.52	0.30	0.00	0.00	0.72	1.539	0.432
12	0.90	0.15	0.13	0.35	0.22	0.97	1.27	0.86	0.38	0.40	0.00	0.00	0.22	1.539	0.432
13	0.75	0.70	0.60	0.36	0.22	0.97	1.27	0.90	0.36	0.25	0.05	0.26	0.46	1.539	0.432
14	0.11	0.41	0.33	0.36	0.21	0.94	0.70	0.91	0.36	0.25	0.24	0.51	0.85	6.636	0.666
15	0.63	0.41	0.60	0.00	0.00	0.00	0.40	0.26	0.12	0.00	0.00	0.00	0.00	6.636	0.666
16	0.71	0.41	0.27	0.70	0.15	0.52	0.52	0.50	0.00	0.00	0.00	0.00	0.00	6.636	0.666
17	0.90	0.41	0.33	0.36	0.21	0.94	0.70	0.91	0.36	0.25	0.24	0.51	0.85	6.636	0.666
18	0.75	0.41	0.60	0.00	0.00	0.00	0.40	0.26	0.12	0.00	0.00	0.00	0.00	6.636	0.666
19	0.63	0.41	0.27	0.70	0.15	0.52	0.52	0.50	0.00	0.00	0.00	0.00	0.00	6.636	0.666
20	0.90	0.41	0.33	0.36	0.21	0.94	0.70	0.91	0.36	0.25	0.24	0.51	0.85	6.636	0.666
21	0.75	0.41	0.60	0.00	0.00	0.00	0.40	0.26	0.12	0.00	0.00	0.00	0.00	6.636	0.666
22	0.63	0.41	0.27	0.70	0.15	0.52	0.52	0.50	0.00	0.00	0.00	0.00	0.00	6.636	0.666
23	0.90	0.41	0.33	0.36	0.21	0.94	0.70	0.91	0.36	0.25	0.24	0.51	0.85	6.636	0.666
24	0.75	0.41	0.60	0.00	0.00	0.00	0.40	0.26	0.12	0.00	0.00	0.00	0.00	6.636	0.666
25	0.63	0.41	0.27	0.70	0.15	0.52	0.52	0.50	0.00	0.00	0.00	0.00	0.00	6.636	0.666
26	0.90	0.41	0.33	0.36	0.21	0.94	0.70	0.91	0.36	0.25	0.24	0.51	0.85	6.636	0.666
27	0.75	0.41	0.60	0.00	0.00	0.00	0.40	0.26</							

[illegible]

May.

[illegible]

June

[illegible]

11011

[illegible]

1000

[illegible]

[illegible][illegible]

Oct.

[illegible][illegible]

Table 3 (2)

Reduced Diurnal Variation of the Water Head at Matoishi (cm)
(from June 1940 to May 1941)

[illegible]

Apr.

[illegible]

May

[illegible]

00

Run	1	2	6	7	8	9	15	16	17	22	23	26	27	28	Total	Mean
1	0	0.00	0.00	0.05	0.05	0.15	0.05	0.00	0.00	0.00	0.40	0.00	0.20	0.00	0.60	0.114
2	0.25	0.15	0.20	0.05	0.15	0.55	0.15	0.05	0.10	0.50	0.40	0.00	0.20	0.15	1.30	0.229
3	0.00	0.00	0.15	0.20	0.25	0.30	0.15	0.25	0.20	0.60	0.15	0.35	0.35	0.40	4.15	0.266
4	0.55	0.15	0.55	0.75	0.75	0.55	0.55	0.00	0.40	0.40	0.50	0.25	0.45	0.60	5.75	0.407
5	0.00	0.00	0.20	0.05	0.05	0.45	0.05	0.35	0.70	0.50	0.50	0.00	0.45	0.55	7.55	0.539
6	0.00	0.15	1.45	1.00	0.95	0.95	0.95	0.25	0.55	0.30	0.45	0.05	0.25	0.65	8.50	0.607
7	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
8	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
9	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
10	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
11	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
12	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
13	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
14	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
15	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
16	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
17	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
18	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
19	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
20	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
21	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
22	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
23	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952
24	0.00	0.35	1.45	1.00	1.00	1.45	1.45	0.50	0.65	0.05	0.45	0.00	0.15	0.75	13.75	0.952

Nov.

[illegible]

Dec.

Day	Hour	4	5	6	8	9	10	11	12	15	13	17	18	20	21	26	28	29	30	31	Total	Mean
0	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
1	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
2	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
3	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
4	5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
5	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
6	7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
7	8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
8	9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
9	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
10	11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
11	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
12	13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
13	14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
14	15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
15	16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
16	17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
17	18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
18	19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
19	20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
20	21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
21	22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
22	23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
23	24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
24	25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
25	26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
26	27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
27	28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
28	29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
29	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
30	31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
31	32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000

Table 4 (1) Observed Data of the Water Head at Uchionmaki (cm) (from July 1948 to June 1949)

Day	Hour	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	2	119.0	113.0	110.0	108.0	106.0	104.0	102.0	100.0	98.0	96.0	94.0	92.0	90.0	88.0	86.0
2	3	118.0	112.0	109.0	107.0	105.0	103.0	101.0	99.0	97.0	95.0	93.0	91.0	89.0	87.0	85.0
3	4	117.0	111.0	108.0	106.0	104.0	102.0	100.0	98.0	96.0	94.0	92.0	90.0	88.0	86.0	84.0
4	5	116.0	110.0	107.0	105.0	103.0	101.0	99.0	97.0	95.0	93.0	91.0	89.0	87.0	85.0	83.0
5	6	115.0	109.0	106.0	104.0	102.0	100.0	98.0	96.0	94.0	92.0	90.0	88.0	86.0	84.0	82.0
6	7	114.0	108.0	105.0	103.0	101.0	99.0	97.0	95.0	93.0	91.0	89.0	87.0	85.0	83.0	81.0
7	8	113.0	107.0	104.0	102.0	100.0	98.0	96.0	94.0	92.0	90.0	88.0	86.0	84.0	82.0	80.0
8	9	112.0	106.0	103.0	101.0	99.0	97.0	95.0	93.0	91.0	89.0	87.0	85.0	83.0	81.0	79.0
9	10	111.0	105.0	102.0	100.0	98.0	96.0	94.0	92.0	90.0	88.0	86.0	84.0	82.0	80.0	78.0
10	11	110.0	104.0	101.0	99.0	97.0	95.0	93.0	91.0	89.0	87.0	85.0	83.0	81.0	79.0	77.0
11	12	109.0	103.0	100.0	98.0	96.0	94.0	92.0	90.0	88.0	86.0	84.0	82.0	80.0	78.0	76.0
12	13	108.0	102.0	99.0	97.0	95.0	93.0	91.0	89.0	87.0	85.0	83.0	81.0	79.0	77.0	75.0
13	14	107.0	101.0	98.0	96.0	94.0	92.0	90.0	88.0	86.0	84.0	82.0	80.0	78.0	76.0	74.0
14	15	106.0	100.0	97.0	95.0	93.0	91.0	89.0	87.0	85.0	83.0	81.0	79.0	77.0	75.0	73.0
15	16	105.0	99.0	96.0	94.0	92.0	90.0	88.0	86.0	84.0	82.0	80.0	78.0	76.0	74.0	72.0
16	17	104.0	98.0	95.0	93.0	91.0	89.0	87.0	85.0	83.0	81.0	79.0	77.0	75.0	73.0	71.0
17	18	103.0	97.0	94.0	92.0	90.0	88.0	86.0	84.0	82.0	80.0	78.0	76.0	74.0	72.0	70.0
18	19	102.0	96.0	93.0	91.0	89.0	87.0	85.0	83.0	81.0	79.0	77.0	75.0	73.0	71.0	69.0
19	20	101.0	95.0	92.0	90.0	88.0	86.0	84.0	82.0	80.0	78.0	76.0	74.0	72.0	70.0	68.0
20	21	100.0	94.0	91.0	89.0	87.0	85.0	83.0	81.0	79.0	77.0	75.0	73.0	71.0	69.0	67.0
21	22	99.0	93.0	90.0	88.0	86.0	84.0	82.0	80.0	78.0	76.0	74.0	72.0	70.0	68.0	66.0
22	23	98.0	92.0	89.0	87.0	85.0	83.0	81.0	79.0	77.0	75.0	73.0	71.0	69.0	67.0	65.0
23	24	97.0	91.0	88.0	86.0	84.0	82.0	80.0	78.0	76.0	74.0	72.0	70.0	68.0	66.0	64.0
24	25	96.0	90.0	87.0	85.0	83.0	81.0	79.0	77.0	75.0	73.0	71.0	69.0	67.0	65.0	63.0

Sep.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	67.6	69.0	66.5	66.2	65.0	62.2	63.2	63.3	61.0	61.2	62.9	89.2	83.7	87.6	87.6
2	67.5	69.1	66.4	66.1	65.0	62.1	63.1	63.2	61.1	61.3	62.9	89.3	83.8	87.7	87.7
3	67.4	69.0	66.3	66.0	64.8	62.0	63.0	63.1	61.0	61.2	62.8	89.4	83.9	87.8	87.8
4	67.3	68.9	66.2	65.9	64.7	61.9	62.9	63.0	60.8	61.1	62.7	89.5	84.0	87.9	87.9
5	67.2	68.8	66.1	65.8	64.6	61.8	62.8	62.9	60.7	61.0	62.6	89.6	84.1	88.0	88.0
6	67.1	68.7	66.0	65.7	64.5	61.7	62.7	62.8	60.6	60.9	62.5	89.7	84.2	88.1	88.1
7	67.0	68.6	65.9	65.6	64.4	61.6	62.6	62.7	60.5	60.8	62.4	89.8	84.3	88.2	88.2
8	66.9	68.5	65.8	65.5	64.3	61.5	62.5	62.6	60.4	60.7	62.3	89.9	84.4	88.3	88.3
9	66.8	68.4	65.7	65.4	64.2	61.4	62.4	62.5	60.3	60.6	62.2	90.0	84.5	88.4	88.4
10	66.7	68.3	65.6	65.3	64.1	61.3	62.3	62.4	60.2	60.5	62.1	90.1	84.6	88.5	88.5
11	66.6	68.2	65.5	65.2	64.0	61.2	62.2	62.3	60.1	60.4	62.0	90.2	84.7	88.6	88.6
12	66.5	68.1	65.4	65.1	63.9	61.1	62.1	62.2	60.0	60.3	61.9	90.3	84.8	88.7	88.7
13	66.4	68.0	65.3	65.0	63.8	61.0	62.0	62.1	59.9	60.2	61.8	90.4	84.9	88.8	88.8
14	66.3	67.9	65.2	64.9	63.7	60.9	61.9	62.0	59.8	60.1	61.7	90.5	85.0	88.9	88.9
15	66.2	67.8	65.1	64.8	63.6	60.8	61.8	61.9	59.7	60.0	61.6	90.6	85.1	89.0	89.0
16	66.1	67.7	65.0	64.7	63.5	60.7	61.7	61.8	59.6	59.9	61.5	90.7	85.2	89.1	89.1
17	66.0	67.6	64.9	64.6	63.4	60.6	61.6	61.7	59.5	59.8	61.4	90.8	85.3	89.2	89.2
18	65.9	67.5	64.8	64.5	63.3	60.5	61.5	61.6	59.4	59.7	61.3	90.9	85.4	89.3	89.3
19	65.8	67.4	64.7	64.4	63.2	60.4	61.4	61.5	59.3	59.6	61.2	91.0	85.5	89.4	89.4
20	65.7	67.3	64.6	64.3	63.1	60.3	61.3	61.4	59.2	59.5	61.1	91.1	85.6	89.5	89.5
21	65.6	67.2	64.5	64.2	63.0	60.2	61.2	61.3	59.1	59.4	61.0	91.2	85.7	89.6	89.6
22	65.5	67.1	64.4	64.1	62.9	60.1	61.1	61.2	59.0	59.3	60.9	91.3	85.8	89.7	89.7
23	65.4	67.0	64.3	64.0	62.8	60.0	61.0	61.1	58.9	59.2	60.8	91.4	85.9	89.8	89.8
24	65.3	66.9	64.2	63.9	62.7	59.9	60.9	61.0	58.8	59.1	60.7	91.5	86.0	89.9	89.9
25	65.2	66.8	64.1	63.8	62.6	59.8	60.8	60.9	58.7	59.0	60.6	91.6	86.1	90.0	90.0
26	65.1	66.7	64.0	63.7	62.5	59.7	60.7	60.8	58.6	58.9	60.5	91.7	86.2	90.1	90.1
27	65.0	66.6	63.9	63.6	62.4	59.6	60.6	60.7	58.5	58.8	60.4	91.8	86.3	90.2	90.2
28	64.9	66.5	63.8	63.5	62.3	59.5	60.5	60.6	58.4	58.7	60.3	91.9	86.4	90.3	90.3
29	64.8	66.4	63.7	63.4	62.2	59.4	60.4	60.5	58.3	58.6	60.2	92.0	86.5	90.4	90.4
30	64.7	66.3	63.6	63.3	62.1	59.3	60.3	60.4	58.2	58.5	60.1	92.1	86.6	90.5	90.5

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
63.3	61.7	59.3	57.5	55.5	53.6	53.0	56.6	102.8	53.6	55.0	100.0	56.6	57.2	54.0
63.6	60.6	58.1	56.2	54.2	52.3	51.7	56.9	100.0	53.9	55.3	99.9	56.9	57.5	54.2
63.7	60.6	58.1	56.2	54.2	52.3	51.7	56.9	97.0	53.9	55.3	99.9	56.9	57.5	54.2
63.4	59.7	57.2	55.2	53.2	51.3	50.7	56.6	91.5	53.6	55.0	99.6	56.6	57.2	53.9
62.7	59.0	56.5	54.5	52.5	50.6	50.0	56.0	92.8	53.9	55.3	99.9	56.9	57.5	54.0
62.1	58.4	55.9	53.9	51.9	50.0	49.4	55.4	90.0	53.6	55.0	99.6	56.6	57.2	53.6
61.6	57.9	55.4	53.4	51.4	49.5	48.9	54.9	89.0	53.3	54.7	99.3	56.3	56.9	53.3
61.0	57.3	54.8	52.8	50.8	48.9	48.3	54.3	87.2	53.0	54.4	99.0	56.0	56.6	52.9
61.2	57.5	55.0	53.0	51.0	49.1	48.5	54.5	88.0	53.3	54.7	99.3	56.3	56.9	53.3
61.7	57.8	55.3	53.3	51.3	49.4	48.8	54.8	89.0	53.6	55.0	99.6	56.6	57.2	53.6
61.0	57.1	54.6	52.6	50.6	48.7	48.1	54.1	87.0	53.0	54.4	99.0	56.0	56.6	52.9
61.3	57.4	54.9	52.9	50.9	49.0	48.4	54.4	88.0	53.3	54.7	99.3	56.3	56.9	53.3
61.5	57.6	55.1	53.1	51.1	49.2	48.6	54.6	89.0	53.6	55.0	99.6	56.6	57.2	53.6
60.9	57.0	54.5	52.5	50.5	48.6	48.0	54.0	87.0	53.0	54.4	99.0	56.0	56.6	52.9
61.1	57.2	54.7	52.7	50.7	48.8	48.2	54.2	88.0	53.3	54.7	99.3	56.3	56.9	53.3
61.0	57.1	54.6	52.6	50.6	48.7	48.1	54.1	87.0	53.0	54.4	99.0	56.0	56.6	52.9
61.3	57.4	54.9	52.9	50.9	49.0	48.4	54.4	88.0	53.3	54.7	99.3	56.3	56.9	53.3
61.6	57.7	55.2	53.2	51.2	49.3	48.7	54.7	89.0	53.6	55.0	99.6	56.6	57.2	53.6
61.7	57.8	55.3	53.3	51.3	49.4	48.8	54.8	90.0	53.9	55.3	99.9	56.9	57.5	53.9
61.3	57.4	54.9	52.9	50.9	49.0	48.4	54.4	88.0	53.3	54.7	99.3	56.3	56.9	53.3
61.6	57.7	55.2	53.2	51.2	49.3	48.7	54.7	89.0	53.6	55.0	99.6	56.6	57.2	53.6
61.8	57.9	55.4	53.4	51.4	49.5	48.9	54.9	90.0	53.9	55.3	99.9	56.9	57.5	53.9

Oct.

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	53.7	53.4	53.8	54.2	52.8	53.6	50.7	50.6	50.2	55.7	57.6	51.5	52.9	52.4	52.6
2	53.7	53.1	53.8	54.2	52.7	53.7	50.8	50.8	50.4	57.0	57.0	51.6	52.6	52.3	52.4
3	54.8	53.2	54.2	54.7	53.1	54.1	51.1	51.1	50.4	51.0	57.9	54.0	52.6	51.8	52.4
4	54.6	53.7	54.0	54.8	54.1	53.2	50.4	50.4	49.8	51.6	58.0	53.3	50.9	51.4	51.7
5	54.2	53.8	54.3	54.8	53.8	53.2	50.8	50.3	49.8	52.3	58.0	53.3	50.9	51.4	51.7
6	54.0	53.5	54.1	54.8	53.4	52.2	50.6	50.0	49.6	52.0	58.0	53.3	50.4	51.0	51.0
7	54.2	53.8	54.3	54.8	53.4	52.7	50.6	50.2	49.6	52.0	58.0	53.3	50.4	51.0	51.0
8	53.8	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
9	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
10	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
11	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
12	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
13	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
14	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
15	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
16	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
17	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
18	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
19	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
20	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
21	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
22	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
23	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0
24	53.0	53.3	53.8	54.3	53.1	53.0	50.9	49.9	49.0	52.0	58.0	53.3	50.4	51.0	51.0

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
50.1	50.0	49.1	48.6	49.6	51.0	49.6	49.7	48.0	49.5	49.6	47.5	47.9	47.6	93.0	44.0
49.9	49.8	48.8	48.2	49.6	51.8	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	93.1	42.9
49.4	49.5	48.4	47.9	49.0	51.0	50.0	49.0	48.5	49.0	49.0	47.5	47.9	47.6	93.2	42.4
49.1	49.4	48.4	47.9	49.0	51.0	50.0	49.0	48.5	49.0	49.0	47.5	47.9	47.6	93.3	42.0
49.8	49.8	48.8	48.2	49.6	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	93.4	42.5
50.0	49.9	48.9	48.3	49.7	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	93.5	41.8
49.6	49.6	48.6	48.0	49.1	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	93.6	41.8
49.7	49.7	48.7	48.1	49.2	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	93.7	41.8
49.6	49.6	48.6	48.0	49.1	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	93.8	41.8
49.4	49.4	48.4	47.8	49.0	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	93.9	41.8
49.0	49.0	48.0	47.4	48.6	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.0	41.8
48.4	48.4	47.4	46.8	48.0	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.1	41.8
49.0	49.0	48.0	47.4	48.6	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.2	41.8
50.6	49.9	48.9	48.3	49.7	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.3	41.8
50.7	49.9	48.9	48.3	49.7	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.4	41.8
50.5	49.9	48.9	48.3	49.7	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.5	41.8
50.0	49.4	48.4	47.8	48.2	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.6	41.8
50.4	49.4	48.4	47.8	48.2	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.7	41.8
50.5	49.4	48.4	47.8	48.2	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.8	41.8
51.0	49.4	48.4	47.8	48.2	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	94.9	41.8
51.9	49.4	48.4	47.8	48.2	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	95.0	41.8
50.3	49.4	48.4	47.8	48.2	51.0	49.1	49.8	48.5	49.3	49.5	47.5	47.9	47.6	95.1	41.8

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	40.4	41.5	42.6	50.5	45.7	42.9	42.8	41.2		40.3	39.8	38.2	37.4	37.4	40.0
2	40.1	41.7	42.4	49.48	45.1	42.6	43.3	40.4		40.5	39.8	38.2	37.4	37.9	
3		41.0	41.4	49.2	45.4	46.1	41.6	40.2		40.0	39.8	38.2	37.4	38.5	
4	39.8	41.2	42.1	49.2	42.7	46.3	41.9	40.0		39.4	38.4	37.3	36.4	36.3	
5	38.8	40.1	42.1	49.2	42.7	46.7	42.3	40.0		39.4	38.4	37.3	36.4	36.3	
6	38.4	40.4	42.1	49.2	42.7	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
7	38.4	40.2	42.1	49.2	42.7	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
8	37.7		42.1	45.3	45.3	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
9	37.7		41.8	44.6	45.3	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
10	38.4	39.8	41.8	45.3	41.8	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
11	38.4	39.8	41.8	45.3	41.8	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
12	38.4	39.8	41.8	45.3	41.8	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
13	38.4	39.8	41.8	45.3	41.8	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
14	38.4	39.8	41.8	45.3	41.8	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
15	38.4	39.8	41.8	45.3	41.8	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
16	38.4	39.8	41.8	45.3	41.8	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
17	38.4	39.8	41.8	45.3	41.8	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
18	38.4	39.8	41.8	45.3	41.8	46.4	42.4	40.0		39.4	38.4	37.3	36.4	36.3	
19	39.9	40.4	40.4	50.8	45.9	41.4	41.3	42.0		40.0	40.8	39.7	38.4	37.4	
20	40.6	40.4	51.7	44.2	42.3	43.6	40.5	40.0		40.3	40.1	37.0	37.0	42.2	
21	42.0	41.0	51.6	44.1	42.6	43.7	40.8	41.0		40.6	38.3	39.5	39.8	39.0	
22	43.0	40.8	53.0	44.1	42.2	44.4	40.5	40.5		39.0	39.0	39.0	39.0	42.5	
23	42.4	41.7	52.4	45.1	41.1	42.6	40.1	39.8		38.9	37.8	39.0	38.2	40.0	
24	41.3	42.2	51.6	45.2	41.5	42.7	40.3	40.6		37.4	35.7	39.0		42.2	

[illegible]

Dec.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	30.7	36.3	31.4	35.9	37.5	31.7	72.1	37.0	38.5	35.0			41.6	42.8	46.7
2	29.9	37.4	32.7	33.3	36.3	31.9	69.2	38.8	37.7	25.7	39.4		41.7	42.6	46.5
3	30.6	37.5	32.8	33.4	36.4	31.3	61.0	39.1	37.0	27.7	39.4		41.7	42.6	46.5
4	32.5	38.5	34.7	35.5	37.5	31.4	52.4	38.8	38.5	28.1			41.6	41.9	46.3
5	30.4	37.4	32.7	33.5	36.4	31.7	54.7	39.3	38.1	29.4			41.6	42.4	46.3
6	30.9	37.5	32.8	33.6	36.5	31.8	48.8	38.7	37.1	30.4			41.8	42.4	46.1
7	29.4	36.8	32.1	33.3	36.2	31.2	48.9	38.6	37.1	30.4			42.2	42.5	45.6
8	30.6	37.5	32.8	33.6	36.2	31.4	48.9	38.6	37.1	30.4			42.2	42.5	45.6
9	29.8	37.0	32.9	33.5	36.4	31.0	47.0	38.8	37.1	30.4			42.2	42.5	45.6
10	33.5	37.0	32.9	33.5	36.4	31.0	46.0	38.8	37.1	30.4			42.2	42.5	45.6
11	33.4	37.3	33.6	33.6	35.0	32.5	43.8	38.8	37.1	30.4			42.2	42.5	45.6
12	34.2	37.3	33.6	34.6	35.0	32.5	39.5	34.4	34.4	41.0			42.2	42.5	44.1
13	33.7	33.5	26.3	36.4	33.9	30.8	39.8	35.8	34.2	42.8			41.7	43.3	44.5
14	34.0	33.5	26.3	36.4	33.9	30.8	40.5	35.4	35.0	42.0			41.8	43.7	44.9
15	35.3	33.5	24.4	36.4	33.9	30.8	39.4	38.4	34.4	40.6			42.2	44.1	44.9
16	35.5	33.7	24.4	36.4	33.9	30.8	39.4	38.4	34.4	41.0			42.2	44.1	44.9
17	36.1	33.3	25.2	36.4	33.9	30.8	39.4	38.4	34.4	41.0			42.2	44.1	44.9
18	36.1	33.3	25.2	36.4	33.9	30.8	39.4	38.4	34.4	41.0			42.2	44.1	44.9
19	35.9	33.3	25.2	36.4	33.9	30.8	39.4	38.4	34.4	41.0			42.2	44.1	44.9
20	34.0	33.3	25.2	36.4	33.9	30.8	39.4	38.4	34.4	41.0			42.2	44.1	44.9
21	35.4	33.3	25.2	36.4	33.9	30.8	39.4	38.4	34.4	41.0			42.2	44.1	44.9
22	35.4	33.3	25.2	36.4	33.9	30.8	39.4	38.4	34.4	41.0			42.2	44.1	44.9
23	35.4	33.3	25.2	36.4	33.9	30.8	39.4	38.4	34.4	41.0			42.2	44.1	44.9
24	35.4	33.3	25.2	36.4	33.9	30.8	39.4	38.4	34.4	41.0			42.2	44.1	44.9

[illegible]

Jan.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	76.3	77.4	55.8	53.1	46.1	44.2	36.9	42.6	33.2	40.7	44.2	42.7	42.6	45.4	48.3
2	78.1	76.1	52.1	54.8	46.1	45.3	34.5	44.3	38.6	40.2	43.8	44.4	43.3	45.8	48.4
3	83.2	74.3	50.1	54.4	46.6	40.5	35.2	45.2			42.6	43.6	43.5	46.4	49.9
4	91.8	74.4	51.0	54.9	47.3	41.0	35.6	47.1			41.1	43.7	43.0	46.4	46.8
5	103.6	60.5	49.9	53.4	46.7	41.4	36.1	46.6	38.0	38.1	43.0	44.1	38.0	46.6	47.4
6	122.2	58.7	52.9	48.1	43.7	41.9	36.4	46.8	36.6	40.0	43.7	37.5	44.5	47.1	44.3
7	143.6	57.3	54.6	48.4	46.1	40.0	36.5	47.0	35.5	41.1	43.3		42.1	47.1	42.7
8	155.3	60.6	49.5	50.3	46.3	36.7	35.7	46.0	37.9	41.4	44.5	40.3	41.7	46.9	42.7
9	155.3	58.9	50.1	51.1	46.6	40.0	36.4	46.9	38.5	41.5	44.8	40.5	41.3	46.9	43.0
10	150.6	60.2	57.2	51.9	43.9	38.8	36.7	46.6		40.3	34.3	45.0	41.1	46.8	43.0
11	140.6	59.8	63.4	52.3	45.5	40.5	36.9	47.2			37.1	45.2	41.0	47.7	
12	132.0	57.7	67.1	49.7	45.7		37.1	46.8			37.7	44.9	41.9	48.1	34.5
13	125.5	52.6	65.7	51.0	45.3	38.3	37.3	45.8	40.1	35.1	45.4	42.3	43.1	48.1	
14	120.6	52.1	69.3	52.3	45.7		37.3	47.2	37.6	33.8	43.7	43.3	41.9	48.1	38.5
15	106.0	52.1	72.6	52.4	44.2	40.0	37.3	42.6			43.2	44.6	42.6	49.2	37.7
16	101.7	51.1	65.5	54.1	44.1		37.3	43.7			43.7	44.6	43.4	49.2	37.6
17	100.4	52.2	67.0	54.1	41.9		37.4	43.7			41.7	45.3	43.6	49.5	
18	94.9	53.3	62.3	52.0	40.0	40.5		43.8			44.7	45.7	44.3	50.3	37.5
19	92.2	57.3	62.6	53.2	40.9	40.3	40.3	43.8	40.0	44.7	45.5	44.6	43.7	50.7	
20	88.3	57.5	61.9	51.7	44.1	40.3	42.2	44.9	38.1	44.1	41.8	44.4	44.0	48.3	34.7
21	85.5	56.8	62.0	50.8	45.5	40.8	42.7	45.9		44.7	43.0	45.7	44.0	47.0	35.2
22	88.3	56.9	58.7	45.5	46.0	39.9	44.3	34.6	40.6	45.6	43.5	46.3	44.1	47.3	35.4
23	82.2	57.7	57.3	46.6	46.4		44.1	38.0	43.0	45.9	43.0	42.5	44.4	48.0	35.7
24	79.9	57.0	55.0	45.5	38.3		43.2	38.5	41.6	46.6	42.8	42.1	44.7	47.7	35.6

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
35.7	42.0	35.6	43.9	43.2	42.3	45.8	42.5	45.3	44.0	42.8	45.1	45.1	45.3	46.3	51.3
35.0	41.8	35.3	42.7	41.6	43.1	46.0	43.1	46.3	44.0	42.3	45.0	45.7	44.9	46.3	51.3
35.6	41.9	36.6	41.3	42.2	41.9	46.6	44.0	47.1	44.7	43.7	44.3	46.0	44.4	48.3	50.7
35.9	42.3	38.3	40.8	42.0	41.1	47.4	44.5	47.7	44.7	43.7	44.3	46.6	44.0	50.0	50.5
36.0	42.7	37.7	41.0	41.3	41.1	47.7	44.3	48.1	44.7	43.9	47.1	47.2	44.6	52.2	49.9
36.1	43.2	36.5	40.9		41.9	48.1	45.3	48.1	44.3	41.5	44.6	47.2	44.6	54.3	49.9
36.4	43.3	36.7		38.8	42.3	47.8	45.7	48.3	45.4	44.6	44.5	46.3	41.7	63.5	49.8
36.4	43.4	38.0	40.7	40.0	43.4	45.8	47.9	48.3	46.2	45.0	44.2	45.9	45.0	67.8	49.7
36.8	43.3	38.7	41.3		42.1	45.3	48.5	47.3	44.1	44.1	43.7	46.2	44.5	70.6	49.1
36.4	43.4	38.6	40.9		40.7	45.3	44.5	46.9	45.0	43.7	44.9	46.5	44.0	69.5	44.6
36.1	44.4	38.3	41.1	40.1	41.1	45.2	43.7	46.6	45.0	43.4	43.7	46.8	44.4	68.5	44.1
36.9	45.0	38.4	42.3	41.1	41.2	45.6	44.1	46.1	44.9	43.5	44.2	46.1	44.9	65.1	42.6
36.7	45.1		43.0	41.4	43.3	42.6	43.9	45.4	44.8	43.4	44.3	45.0	45.8	65.5	42.6
37.1	45.2	39.0	43.4	37.9	43.9	42.7	44.8	44.5	43.7	44.6	44.8	44.9	46.3	64.4	42.2
38.8	45.7		44.0	44.4	44.4	43.6	45.1	45.1	43.7	44.7	44.6	43.9	47.0	62.2	41.5
	47.5		44.5	40.7	44.1	42.3	45.7	45.7	42.3	43.3	43.7	41.0	47.7	62.3	40.0
40.1	48.5	40.1	44.3	42.3	43.2	43.2	43.2	45.6	42.4	45.5	43.6	41.6	48.3	61.2	39.7
40.5	48.8	40.7	44.4	43.7	42.6	43.6	43.6	45.6	41.1	45.1	44.8	45.5	47.0	61.0	38.3
41.1	48.7	42.7	45.5	45.3	42.9	44.1	44.1	45.0	42.2	45.3	45.0	45.0	46.6	57.7	38.3
41.0	48.8	45.2	44.1	44.4	44.4	45.3	44.3	45.3	42.2	45.3	45.2	45.1	46.6	55.5	38.8
41.7	47.0	45.2	44.7	44.4	44.7	45.3	43.3	44.1	42.5	44.6	44.6	45.1	45.1	55.3	
41.1	37.0	45.2	44.7	44.8	41.3	45.3	43.4	45.2	42.6	44.6	44.6	45.2	45.2	55.3	38.7
42.1	37.1	44.8	44.9	42.1	45.7	43.1	44.3	43.9	42.8	41.0	44.8	45.5	45.8	53.2	

Feb.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hour															
1		60.6	43.0	48.0	37.3	38.6	38.3	41.3	44.3	40.8	40.9	67.5	59.8	53.2	42.5
2	38.5	58.8	42.0	48.1	38.9		38.6	41.3	44.6	41.7	41.0	67.5	59.7	53.4	40.3
3	38.5	58.4		48.0	36.7	38.2	38.7		43.4	43.1	40.7	68.5	57.6	50.4	
4	37.8	57.0		49.0	36.5	38.3	37.2	40.0	42.8	42.3		67.5	57.3	49.3	
5	38.0	54.9	40.4	47.5	35.7	34.3	36.7		41.8	41.5		66.1	57.4	51.0	38.8
6	37.9	48.1	40.4	47.6	38.4	38.8	37.0		40.2	40.5		68.3	57.5	49.5	38.3
7	38.1	44.1	42.2	47.2	39.0	38.8	37.3	40.7	41.1	40.9	40.3	68.4	57.2	47.3	40.8
8	39.0	42.1	42.2	48.2	37.5	35.1	37.6		43.5	41.5	41.0	67.1	57.5	48.3	41.1
9		41.0	42.2	43.7	40.3	35.9	37.3	41.8	43.0	42.2	40.3	68.9	57.5	50.8	41.7
10		41.3	41.3	44.0	40.4	38.8	37.8	41.5	43.1	41.2	41.7	65.0	55.3	49.0	41.7
11		43.3	41.3	44.6	35.8	35.3	37.9	41.5	42.9	41.1	41.7	65.0	55.3	48.2	42.9
12	42.3	44.7	41.3	43.9	38.6	36.7	38.8		43.1	40.9	43.0	64.5	55.1	47.3	42.4
13	49.0	44.7	42.2	43.5	37.3	37.5	38.2	41.2	42.6	41.0	44.0	65.9	55.1	46.0	43.3
14	58.7	44.3	42.2	43.0	41.1	40.1	38.8	41.1	42.3	41.2	44.0	63.4	55.2	42.9	42.9
15	63.0	45.9	41.4	43.0	40.3	42.6	42.9	41.9	43.1	41.3	44.4	62.2	55.3	42.0	44.1
16	67.3	45.3	43.3	43.7	40.3	43.2	42.9	43.3	43.7	40.6	44.9	61.1	55.1	46.6	47.4
17	67.3	48.9	45.5	42.7	41.7	43.6	43.6	43.7	43.7	40.1	45.6	60.0	55.6	46.5	49.1
18	65.9	42.7	47.1	41.1	42.4	43.1	40.5	43.6	39.9	40.3	44.3	59.5	54.8	44.6	47.7
19	63.4	43.8	48.3	41.5	39.2	41.1	39.9	43.3	40.3	41.0	44.0	57.7	54.8	44.3	47.1
20	64.8	44.7	48.3	41.1	41.1	41.8		43.3	40.3	40.6	54.3	56.8	54.8	44.3	45.9
21	65.2	43.9	47.9	37.1	40.3	40.3	40.5	42.4	39.7	40.0	62.0	58.8	54.2	46.8	45.2
22	69.0	43.8	48.0	34.8	38.1	38.1	41.0	43.6	39.8	39.8	68.0	58.8	54.2	46.8	44.4
23	62.5	43.1	48.1	36.0	40.3	38.0	41.0	44.0	40.0	40.0	68.7	59.9	54.6	43.7	44.8

16	17	18	19	20	21	22	23	24	25	26	27	28
44.8	41.6	40.9	42.8	46.7	45.8	43.0	60.6	66.0	47.7	55.7	43.3	43.0
43.3	41.3	42.8	42.3	45.2	46.4	43.4	60.3	45.6	55.0	55.8	42.0	42.8
43.3	40.4	41.3	42.3	45.0	46.1	43.4	60.0	44.4	52.2	57.7	41.8	40.3
43.5	40.3	42.2	42.5	45.0	45.0	43.3	60.2	42.6	70.0	57.7	42.3	39.5
	39.1	41.4	44.4	46.4	45.2	44.1	61.0	42.6	70.0	57.7	42.3	
	38.3	40.3	44.4	46.4	45.2	44.1	61.0	42.6	70.0	57.7	42.3	39.4
	37.8	40.6	42.2	42.8	45.5	44.4	60.8	40.3	84.8	57.7	42.8	40.2
38.8	40.5		42.5	42.8	46.3	45.4	60.3	38.4	84.8	57.7	42.8	
40.0	40.0		43.5	43.1	45.8	45.6	78.3	38.0	85.0	53.3	41.3	41.4
41.3	40.3		41.5	44.9	44.0	45.0	80.3	37.0	80.0	51.9	42.0	41.1
39.3		40.3	41.7	44.2	44.0	46.2	82.5	37.0	79.0	52.8	43.2	42.3
38.5	40.3		41.8	43.4	43.5	45.7	83.7	34.7	77.3	47.3	43.2	42.0
40.7		40.3	42.7	43.0	43.8	48.7	82.9	34.7	77.3	44.7	44.2	40.6
41.3	41.1		42.4	44.3	43.8	54.2	81.3	37.2	74.0	43.3	42.9	41.1
42.0	42.1	41.7	43.7	43.9	44.4	60.0	78.6	37.2	70.1	44.7	41.2	40.5
42.2	41.7	41.8	43.6	45.8	43.3	63.1	77.1	38.5	87.0	48.3	42.6	
42.6	41.1	45.2	44.7	46.4	43.1	65.1	75.6	38.4	66.4	45.7	42.8	41.0
41.9	41.1	45.3	44.3	46.4	42.9	65.4	73.3	35.9	60.0	45.2	42.6	41.2
42.0	41.3	44.7	46.6	46.4	42.5	64.6	68.2	41.3	60.1	47.3	41.1	
41.7	42.0	44.5	46.6	46.4	44.1	63.3	65.5	43.5	53.1	44.7	42.6	
41.5	41.4	43.9	46.2	45.0	43.5	62.1	66.3	44.2	52.2	45.8	42.2	39.6
			46.2	45.7	43.0	61.6	65.0		44.5	43.6		

Mar.

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1															
2															
3	40.3	38.6	38.7	38.6	38.9	58.6	40.0	36.3	40.3		161.3	56.6	42.6	123.6	104.5
4		40.3	38.9			58.7	38.9	36.2	42.1	40.2	152.7	56.7	42.7	123.6	104.5
5	30.3	41.8		38.7		53.7		36.6	42.7	39.7	147.5	52.0	46.1	142.1	103.0
6		40.3	39.0	38.9	40.4	53.5	38.9	36.2	43.8	40.7	142.3	46.2	43.6	132.1	103.6
7	40.9	38.9	38.2	38.6	44.0	49.6	40.0	36.8	42.4	40.9	139.1	49.1	44.0	121.1	109.1
8	41.3	37.2	38.0		30.5	45.5	39.9	36.0	43.8	40.5	134.3	45.8	46.3	115.1	114.0
9	40.8		38.0	38.9	66.8	45.7		37.9	42.6	40.0	122.0	45.6	42.5	102.3	118.0
10	42.0		38.0		84.5	49.7		36.9	44.3	40.0	118.3	45.3	42.0	100.5	119.0
11	41.0		39.1		98.3	49.0		34.7	44.7	40.0	115.2	50.3	41.5	95.8	116.4
12	41.6	41.3		39.0	120.5	49.1	39.7	34.3	44.0	40.0	109.2	52.3	42.3	91.0	113.7
13		40.6	39.5	39.5	118.0	49.3	39.9	35.0	44.0	40.2	103.3	52.1	39.9	87.5	113.7
14		40.6	39.8	39.5	108.0	41.7	38.7	34.6	43.5	42.8	101.3	50.3	40.5	83.3	116.3
15	40.8	40.1	39.9	38.9	100.5	40.3	38.4	36.3	43.4	54.9	98.0	48.7		83.1	114.4
16	38.6	40.0	38.9		94.7	40.8	38.7	37.8	42.0	103.3	91.5	51.0	39.0	81.4	100.5
17	40.4	41.1	39.4	38.8	88.5	40.7	38.3	39.0	42.9	164.7	86.3	50.6	43.2	81.4	99.7
18	40.2	40.0	38.9	38.7	83.9	40.5	39.1	39.0	42.4	199.7	82.9	47.7	40.6	88.4	96.7
19		40.7			81.1		37.6	38.5	41.8	201.7	73.1	45.8	40.8	90.3	97.1
20	40.6	38.2	40.4	38.8	77.5		36.9	39.1	40.7	183.6	69.0	46.2	42.0	95.1	94.7
21	40.9	40.0			74.8		35.0	39.5	41.5	179.5	70.0	45.8	41.3	100.2	92.2
22	36.6	37.3			70.0		35.0	39.0	40.6	176.1	66.6	43.6	46.1	104.5	88.8
23		39.0	38.5		65.5		35.7	39.5	40.5	173.6	61.7	44.4	52.0	105.7	88.8
24		38.0			61.0	38.9	33.5	40.0	40.7	168.4	59.0	44.5	80.9	103.1	79.1

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
81.8	59.0	71.7	54.6	55.4	62.5	50.4	53.2	54.3	49.8	53.7	47.5	53.9	45.9	45.6	48.1
81.1	57.7	72.9	55.2	51.8	66.0	49.6	52.4	55.2	50.2	55.6	48.8	50.9	45.0	46.7	49.3
78.9	58.2	74.7	56.7	51.9	67.0	48.6	54.4	55.3	51.9	55.8	46.9	50.0	45.2	46.4	50.4
78.1	57.5	71.1	56.7	52.6	66.8	53.3	53.7	55.1	51.9	55.3	48.0	50.5	44.4	45.7	51.6
77.1	53.1	69.4	57.0	51.0	71.6	48.0	52.4	52.3	49.5	56.5	49.5	50.7	43.7	43.0	48.1
77.2	57.7	65.1	55.0	55.6	68.7	47.4	51.9	53.8	50.7	55.7	48.9	47.8	44.2	43.4	48.7
76.8	59.5	64.6	50.3	55.7	70.6	52.3	49.9	51.5	49.3	53.2	48.7	47.0	44.3	47.1	49.7
76.4	60.3	66.7	50.1	57.7	67.0	54.1	50.7	48.3	52.2	53.9	54.6	49.3	44.3	47.0	50.1
75.5	55.1	65.0	50.0	58.0	63.3	51.3	49.3	47.7	51.9	47.7		48.0	43.6	47.9	50.2
75.0	55.8	58.3	56.4	56.1	60.5	51.1	53.2	50.7	50.9	47.6		48.0	43.6	47.9	50.2
73.9	57.1	66.3	58.7	56.3	59.2	51.6	51.5	50.4	50.1	48.1	52.7	45.9	43.1	48.1	45.7
71.4	57.2	65.6	57.8	50.1	60.5	48.7	49.8	50.3	49.1	47.8	53.5	44.7	47.0	48.0	49.1
72.6	58.0	64.7	53.5	55.0	59.8	48.6	52.0	50.4	49.2	47.9	52.9	44.4	45.0	49.8	50.5
71.1	54.7	62.2	53.0	51.2	58.8	49.7	52.0	52.3	51.7	50.1	58.5	44.5	44.3	49.8	50.5
70.3	52.4	62.2	52.4	53.5	56.6	48.7	52.6	52.0	50.5	50.7	59.3	49.8	45.5	49.7	50.5
74.3	55.6	62.1	53.3	51.2	56.3	48.2	49.7	52.5	49.6	52.6	58.9	50.2	44.5	50.0	48.7
73.1	59.9	61.1	56.3	51.4	57.3	48.3	25.5	53.2	53.7	50.3	55.5	47.7	46.6	49.5	51.1
72.1	61.1	57.7	53.3	55.2	48.6	49.7	53.3	52.2	53.9	47.7	55.1	45.5	47.5	49.5	50.5
57.4	58.0	57.2	53.2	58.3	50.9	49.3	53.6	49.9	54.1	50.7	56.2	46.5	46.2	49.6	48.8
57.4	58.0	57.2	53.2	58.3	50.9	49.3	53.6	49.9	54.1	50.7	56.2	46.5	46.2	49.6	48.8
59.8	62.5	55.0	55.6	60.5	50.0	50.9	54.0	50.5	54.8	45.8	53.3	44.1	47.6	49.9	48.6
60.6	67.5	55.3	55.3	60.8	50.9	51.9	54.5	49.3	54.9	48.3	54.1	44.5	45.5	49.9	49.3
53.4	68.8	55.0	54.2	60.3	51.4	52.5	55.0	49.7	53.9	50.6	53.7	44.4	44.6	47.7	49.0

Apr.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hour															
1	51.1	50.8	48.0	55.3	46.8	50.5	43.6	44.9	42.2	42.4	53.3	48.1	49.1	50.2	52.3
2	50.3	49.5	48.4	56.3	47.3	50.9	45.1	44.7	43.5	46.2	53.8	47.9	49.0	50.6	52.7
3	48.4	49.9	49.9	55.5	46.5	50.9	43.0	45.5	44.3	46.5	53.4	48.3	48.4	50.8	52.8
4	48.1	49.8	51.1	54.9	46.5	51.3	44.3	45.6	44.9	45.7	54.2	48.9	48.5	51.0	52.6
5	46.6	48.4	50.8	54.0	47.4	50.0	44.5	45.1	45.5	45.3	55.2	49.3	48.8	51.4	52.6
6	47.0	48.4	52.3	50.7	44.3	49.9	45.5	46.1	44.9	45.8	54.2	46.3	47.8	50.3	51.5
7	50.5	49.0	51.8	52.9	45.7	49.7	45.4	47.8	45.5	46.0	54.7	45.9	47.4	50.3	51.9
8	47.5	48.4	53.5	51.1	49.0	49.6	48.0	46.3	46.0	46.3	54.7	44.8	48.6	50.6	52.1
9	51.0	50.1	52.7	54.0	48.7	51.0	48.2	47.3	48.2	45.9	55.0	46.5	47.2	50.6	52.3
10	50.5	49.9	53.7	51.7	49.2	47.2	46.8	46.8	46.3	44.9	54.9	47.1	48.0	50.7	51.8
11	50.4	48.6	50.2	50.1	48.3	48.8	44.6	47.4	46.2	44.3	54.7	48.4	48.3	50.9	51.0
12	48.6	46.3	50.2	48.4	46.9	48.8	44.6	47.4	46.2	44.3	54.7	49.3	47.8	50.3	53.6
13	49.8	46.5	50.7	47.8	47.2	47.9	45.0	47.1	47.0	43.3	55.4	49.9	49.5	51.2	53.9
14	47.4	48.8	51.1	47.5	46.5	47.7	44.9	47.5	46.4	45.2	53.6	49.8	50.0	51.5	54.2
15	50.9	48.2	52.3	45.3	47.0	44.7	47.7	47.4	43.3	47.1	53.6	50.5	50.4	50.3	54.4
16	49.5	47.0	52.5	46.8	48.0	44.8	42.9	47.5	46.7	46.8	54.7	49.9	49.9	51.4	54.5
17	50.2	46.9	53.6	50.9	49.0	42.9	44.2	47.0	45.1	45.8	55.4	49.6	50.2	51.8	54.4
18	50.2	46.9	55.2	50.5	49.2	43.8	44.6	47.7	45.9	46.0	54.0	48.3	50.3	52.2	54.3
19	49.6	46.7	56.6	50.9	48.4	43.0	43.9	43.8	46.5	48.0	54.8	48.0	49.5	51.3	54.0
20	44.4	47.2	57.8	48.9	47.8	44.0	43.5	46.1	46.1	48.5	54.7	47.9	49.9	51.9	54.2
21	47.2	47.5	57.4	47.8	48.4	44.2	43.8	42.3	45.7	49.1	50.0	48.0	49.4	52.0	53.5
22	47.5	48.9	57.3	51.0	48.9	44.8	44.9	42.1	45.9	49.3	50.0	48.3	49.3	52.0	53.5
23	48.5	49.6	56.5	51.2	49.2	45.8	44.5	41.5	46.7	51.5	48.4	48.5	49.4	52.1	54.0

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
54.8	55.9	58.7	65.6	66.2	37.0	39.5	45.3	57.2	42.3	36.3	171.6	62.5	41.3	34.7
54.9	55.8	58.7	65.7	64.7	37.3	40.0	44.1	57.3	40.0	35.3	161.7	61.5	38.9	40.1
54.9	55.3	58.7	65.9	64.2	36.4	40.4	44.0	56.7	40.1	36.0	154.6	61.3	42.7	40.8
55.1	55.6	58.9	68.8	64.4	37.4	40.9	45.0	54.1	40.7	35.8	148.6	60.7	43.1	42.1
55.3	55.7	59.0	69.3	62.8	38.0	41.5	46.4	53.3	41.0	37.0	141.9	60.4	43.2	42.1
55.6	55.7	59.1	69.9	58.7	37.5	42.5	48.9	49.1	40.7	38.0	135.7	59.5	44.6	42.7
55.4	55.4	59.3	70.4	58.2	38.0	43.5	40.6	50.2	40.2	38.7	130.8	58.9	43.0	43.5
55.6	55.7	59.5	70.4	57.3	38.3	44.0	44.2	49.0	42.3	39.1	127.6	58.7	41.0	42.4
55.8	55.7	60.0	70.5	37.3	38.2	44.5	43.5	46.8	41.7	41.0	126.7	54.1	41.1	43.0
55.5	55.4	60.3	71.0	40.2	38.6	45.6	49.0	47.5	42.7	41.5	121.5	50.3	41.0	45.5
55.4	55.4	60.9	72.4	40.2	38.6	45.7	51.7	45.5	41.8	42.4	117.8	45.5	40.7	45.3
55.6	55.4	61.4	72.3		39.1	45.5	55.5	46.1		44.1	107.7	50.2	41.1	46.3
55.6	55.4	61.4	71.8	37.9		46.5	57.3	47.3	40.6	48.0	103.9	51.8	41.5	49.7
55.6	55.4	61.4	71.0	37.9	40.0	46.5	58.3	45.5		54.7	102.4	51.1	42.3	51.7
55.6	55.4	61.4	70.3	35.8		46.5	60.0	47.5		72.8	94.7	41.9	42.0	53.5
55.8	55.4	61.4	69.5	35.5		46.5	61.2	45.1		105.0	92.4	44.1	41.7	64.1
55.4	55.4	61.4	68.0	36.5		44.5	60.6	42.8		138.3	83.6	43.7		78.3
55.5	55.4	61.4	68.0	36.5		44.5	60.1	43.0		165.0	63.6	43.9		93.2
55.6	55.4	61.4	67.7	37.0		44.7	58.9	43.1		186.1	63.0	43.1	42.8	109.3
55.6	55.4	61.4	67.5	37.2		44.7	58.3	43.0		191.4	58.6	38.7	43.0	116.5
55.6	55.4	61.4	67.3	37.3		45.2	57.5	41.8		187.7	60.6	40.7	42.4	123.2
55.8	55.4	61.4	67.3	37.3						179.7	62.8	41.0	42.9	126.5

May

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	123.3	61.4	44.1	45.3	43.9	43.5	65.2	50.1	42.3	43.1	41.5	42.9	90.3	51.7	50.0
2	120.4	58.0	47.0	45.0	44.2	45.3	63.3	50.9	44.8	43.7	40.2	43.9	88.0	51.0	52.9
3	116.4	56.7	46.0	44.0	44.2	45.3	60.1	50.3	43.0	43.2	41.0	43.9	86.3	50.3	52.1
4	113.1	56.0	45.0	43.1	44.0	45.3	57.7	52.2	47.3	43.0	41.2	45.2	85.0	50.0	52.7
5	104.7	60.9	47.0	45.3	44.6	45.3	59.1	54.8	48.0	43.0	44.0	44.0	83.9	53.2	47.0
6	102.5	60.1	48.8	45.9	44.6	45.3	59.0	58.0	45.5	43.5	42.8	44.0	82.1	46.9	50.5
7	101.8	59.8	48.0	45.3	45.1	45.3	57.2	53.7	43.7	43.7	42.2	44.5	81.1	45.7	53.0
8	102.9	58.7	44.0	45.3	42.2	45.3	57.4	49.0	44.9	45.3	42.7	43.0	89.6	49.8	48.4
9	100.5	51.3	48.1	46.3	43.3	45.3	59.8	48.0	43.7	45.0	43.8	47.3	79.5	55.0	47.7
10	97.8	50.8	48.9	45.2	44.8	45.3	58.2	47.0	45.1	41.9	45.0	48.1	77.0	58.0	48.8
11	96.7	50.9	49.2	45.7	44.2	45.3	55.5	50.8	45.9	44.7	42.9	49.5	73.9	59.5	52.2
12	94.5	52.5	50.0	46.5	46.0	45.3	55.1	48.5	40.9	54.3	42.9	49.9	72.9	60.0	51.7
13	92.0	55.3	48.7	45.6	46.8	45.3	55.0	50.0	47.0	41.7	42.1	49.7	68.3	61.1	51.3
14	88.1	51.8	49.0	46.0	47.8	45.3	55.0	46.0	44.6	44.1	42.6	51.3	61.1	61.8	51.7
15	84.9	55.3	49.3	46.0	48.0	45.3	55.4	50.0	44.6	44.1	42.7	54.0	65.2	61.9	51.7
16	82.0	52.5	46.7	46.0	47.7	45.3	54.3	51.0	44.0	44.1	43.7	55.8	63.3	60.0	51.3
17	77.3	52.5	49.3	45.5	47.2	45.3	54.3	48.3	44.4	44.1	43.9	57.4	60.0	60.0	50.5
18	76.4	49.4	48.5	45.4	47.5	45.3	53.7	44.1	41.4	43.4	44.0	57.4	59.0	54.4	49.3
19	75.0	53.1	45.7	44.3	44.3	45.3	53.8	44.7	41.3	42.9	45.5	57.4	57.4	54.4	48.8
20	58.7	45.6	48.8	42.3	44.9	45.3	52.0	46.3	41.3	40.3	45.4	90.9	53.0	51.4	49.1
21	58.5	48.9	44.5	43.9	42.7	45.3	50.0	47.2	42.5	40.6	44.1	94.9	53.0	48.8	47.6
22	61.8	45.5	46.0	42.7	45.3	44.9	51.1	46.9	42.9	41.3	41.3	91.0	54.2	49.0	48.2

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
49.3	51.7	56.4	59.0	212.7	138.2	71.0	219.4	174.3	109.9	68.8	63.5	61.1	103.1	66.7	67.0
49.4	51.9	56.6	59.6	229.5	130.3	67.9	220.5	173.7	105.0	70.2	64.9	63.9	100.0	73.3	67.2
49.5	51.7	55.9	60.3	241.7	118.3	68.5	222.5	171.3	104.0	70.9	64.8	68.8	90.0	73.6	67.6
49.6	51.0	58.0	60.3	241.7	118.2	64.7	234.0	172.3	103.9	71.4	64.8	73.3	92.7	72.0	67.0
50.0	52.4	58.5	59.6	232.1	118.3	68.8	249.5	169.0	101.8	71.0	58.5	80.5	88.8	74.0	68.0
50.1	52.3	59.3	59.4	270.3	112.5	60.1	265.0	148.7	96.3	71.0	61.5	89.5	88.8	74.2	68.3
50.2	52.6	60.3	54.7	279.3	102.0	64.8	275.7	131.9	98.3	71.6	61.5	104.3	87.7	74.2	68.3
51.1	52.3	61.3	56.8	283.2	99.0	63.8	280.8	128.5	97.8	70.0	61.5	120.3	80.7	74.7	68.3
51.2	52.4	62.2	57.7	279.5	93.5	61.2	286.8	125.7	89.1	69.2	61.9	142.3	78.7	75.5	68.2
51.3	52.2	62.2	56.3	272.1	95.3	62.0	290.0	105.5	77.5	67.1	62.3	175.5	73.7	74.7	68.5
51.4	51.5	62.0	55.4	264.5		62.0	288.7	106.0	78.5	67.9	61.9	177.0	74.0	73.8	68.4
51.5	53.0	61.9	55.4	254.4	89.8	60.3	292.5	107.7	80.1	68.6	62.5	171.6	72.5	75.2	67.8
51.6	53.7	61.4	53.8	244.4	86.2	60.3	288.6	106.8	75.7	69.0	62.4	160.8	70.7	75.2	69.7
51.7	54.0	62.0	55.9	209.8	75.4	60.5	280.7	108.7	73.7	69.1	62.6	152.5	71.7	74.1	65.6
51.8	54.1	61.2	55.9	194.0	80.2	58.5	268.7	113.3	73.7	65.8	60.7	140.0	69.9	75.0	65.3
51.9	54.5	60.2	55.8	180.5	84.0	58.3	254.1	115.0	74.5	63.5	61.9	133.5	68.7	74.8	65.9
51.0	53.7	60.3	58.3	168.8	81.9	50.5	240.0	119.0	75.1	65.0	59.8	125.8	70.7	72.2	66.2
51.1	54.0	60.3	64.3	158.8	87.0	59.0	228.8	120.6	74.5	65.0	61.0	121.5	71.7	71.1	66.2
51.2	53.7	59.7	75.6	154.4	86.5	63.1	218.0	120.4	74.5	65.5	61.4	115.0	70.0	62.9	62.2
51.3	54.0	59.0	100.0	149.3	87.6	71.8	205.0	118.2	73.5	65.5	59.3	113.0	69.9	62.1	62.2
51.4	55.0	58.4	133.1	145.3	85.4	107.3	195.5	116.6	73.2	64.4	61.1	107.1	67.6	66.7	63.4
51.5	55.5	58.4	141.1	141.1	88.8	155.0	190.1	113.3	71.9	64.4	61.6	106.5	69.4	67.0	64.6
51.6	55.2	58.6	186.7	137.1	71.1	212.0	188.1	111.6	72.7	64.4	62.8	105.1	70.0	65.3	60.9

June

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	63.8	64.3	64.4	74.3	61.6	53.0	56.0	124.1	78.2	68.3	63.0		56.1	58.0	57.8
2	64.7	65.2	65.1	73.9	61.7	53.2	56.7	123.3	76.8	67.5	63.4		55.9	58.4	58.4
3	65.9	67.0	67.1	73.7	61.5	53.3	57.8	114.0	77.0	66.5	62.9		57.0	58.7	58.3
4	66.6	67.8	67.8	75.2	61.7	53.7	58.0	110.3	77.3	66.1	61.8		57.0	59.5	58.9
5	65.8	66.1	66.1	71.7	60.9	53.1	55.1	109.2	76.7	67.3	62.6		56.8	59.5	57.8
6	65.4	66.4	66.5	72.5	61.3	53.1	55.1	107.3	77.3	67.3	62.9		57.7	58.7	58.5
7	65.8	67.2	67.2	71.5	61.3	53.1	55.1	103.1	77.3	67.3	62.6		57.7	58.8	58.5
8	65.7	67.9	67.9	72.0	60.9	53.6	55.7	101.1	77.3	67.3	62.6		57.7	59.0	59.0
9	63.4	66.5	66.5	72.0	60.0	53.7	55.7	97.2	77.3	67.3	62.6		57.7	59.0	59.0
10	63.4	66.5	66.5	71.9	58.8	53.9	55.7	92.7	77.3	67.3	62.6		57.7	59.0	59.0
11	63.2	66.5	66.5	71.9	58.8	53.9	55.7	92.7	77.3	67.3	62.6		57.7	59.0	59.0
12	63.2	66.5	66.5	71.9	58.8	53.9	55.7	92.7	77.3	67.3	62.6		57.7	59.0	59.0
13	62.6	65.8	65.8	70.7	59.0	53.7	55.7	87.0	77.3	67.3	62.6		57.7	59.0	59.0
14	62.3	65.4	65.4	70.7	59.0	53.7	55.7	87.0	77.3	67.3	62.6		57.7	59.0	59.0
15	62.3	65.4	65.4	70.7	59.0	53.7	55.7	87.0	77.3	67.3	62.6		57.7	59.0	59.0
16	61.1	65.9	65.9	68.0	57.0	53.6	55.6	80.1	77.3	67.3	62.6		57.7	59.0	59.0
17	62.7	66.5	66.5	68.8	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
18	64.8	69.4	69.4	74.4	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
19	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
20	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
21	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
22	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
23	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
24	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
25	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
26	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
27	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
28	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
29	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0
30	65.4	70.0	70.0	75.0	55.5	53.8	55.8	79.4	77.3	67.3	62.6		57.7	59.0	59.0

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
64.0	116.4	69.6	301.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
5.7	105.3	70.2	311.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
7.7	99.3	71.4	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
8.0	105.1	71.1	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
10.7	104.5	72.0	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
12.7	97.6	72.8	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
13.6	89.2	73.2	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
20.0	90.0	73.2	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
22.9	88.8	73.2	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
23.2	89.1	73.2	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
23.7	89.1	73.2	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
24.8	81.3	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
22.8	80.0	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
21.7	79.0	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
20.6	76.3	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
19.5	73.4	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
18.5	73.4	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
16.6	73.4	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
15.4	73.4	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
13.0	73.4	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
12.6	72.2	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
12.0	71.1	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
11.9	71.1	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1
11.8	70.0	74.5	312.3	211.5	239.4	147.7	115.1	115.1	115.1	98.8	109.8	111.2	104.1	305.1

Taale 4 (2)

Observed Data of the Water Head at Matoishi (cm)

(from June 1940 to May 1941)

June

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	15.0	22.0	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		93.5	78.3	35.5	33.0
2	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		101.5	77.3	35.5	33.0
3	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		102.5	77.3	35.5	33.0
4	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		101.5	77.3	35.5	33.0
5	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		101.5	77.3	35.5	33.0
6	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		98.5	77.3	35.5	33.0
7	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		99.5	77.3	35.5	33.0
8	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		99.5	77.3	35.5	33.0
9	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		81.5	77.3	35.5	33.0
10	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
11	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
12	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
13	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
14	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
15	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
16	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
17	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
18	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
19	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
20	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
21	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
22	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
23	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0
24	15.0	21.1	15.0	13.8	10.0	27.5	25.0	19.8	16.5	14.0		77.5	77.3	35.5	33.0

July

16	17	18	19	20	21	22	23	22	23	24	25	26	29	30	31
21.5	19.3	99.5	66.0	61.5	30.5	22.0	65.0		50.5	27.0	33.0	23.0		22.5	27.5
21.5	13.3	123.5	69.0	60.5	30.3	22.5	65.0		50.5	33.5	33.0	23.0		22.0	27.5
21.5	13.3	140.5	77.5	59.0	30.3	22.5	65.0		50.5	33.5	33.0	23.0		22.0	27.5
21.5	19.0	148.0	87.3	57.5	30.0	24.5	67.0		50.5	33.5	33.0	23.5		22.5	27.5
21.5	19.0		99.3	55.8	30.0	25.5	67.0		50.5	37.0	33.0	23.0		23.0	28.0
21.5	19.5		107.5	55.3	30.0	27.3	67.0		50.5	37.0	33.0	24.0		24.0	28.0
21.5	19.5		107.5	55.3	30.0	31.5	65.0		50.5	33.5	33.0	24.5		24.0	28.5
21.5	20.0	144.5	111.5	55.3	26.0	31.5	61.8	98.0	48.5	37.0	32.5		17.5	24.5	29.5
21.5	20.0	129.0	111.5	51.5	26.3	45.5		98.5	48.0	37.0	32.0		13.0	24.5	29.5
21.5	20.3	121.0	110.5	51.0	26.3	55.5		98.5	47.0	37.0	31.5		40.0	24.0	30.0
21.5	20.5	123.5	108.5	49.5	26.3	65.5		98.0	43.0	33.5	31.5		40.5	24.0	30.0
21.5	20.3	117.3	104.5	47.5	26.3	70.8		76.0	41.5	36.0	31.0		40.0	23.5	29.5
22.0	20.0	105.5	101.5	45.3	26.0	73.3		72.5	42.0	35.5	30.5		39.5	23.0	29.5
22.0	19.8	98.5	97.5	43.0	25.5	73.5		70.0	40.0	35.0	30.0		39.0	23.5	29.0
22.0	19.3	91.5	92.8	40.8	25.3	73.3		67.0	41.0	35.0	29.0		38.5	23.0	28.5
22.0	19.0	84.5	88.5	39.0	24.3	71.5		64.5	40.5	35.0	28.0		38.0	23.0	28.0
22.0	18.3	78.3	83.3	37.0	23.5	69.0		62.0	40.0	34.0	27.0		37.0	23.0	27.5
22.0	18.0	71.0	78.5	35.3	23.0	67.0		60.0	39.5	33.0	26.5		37.0	23.5	27.0
22.0	17.3	69.5	76.8	34.8	22.3	65.0		59.0	38.5	33.0	25.5		36.5	23.0	26.5
22.0	17.0	67.0	73.8	33.8	22.0	64.0			37.5	32.5	25.0		36.5	23.0	26.5
22.0	16.3	63.5	70.8	32.3	22.0	64.0			37.5	32.5	24.0		34.0	23.0	26.0
22.0	16.0	61.0	68.3	31.3	21.5	63.3			37.5	32.5	23.5		33.5	23.0	25.5
22.0	15.3	58.5	65.0	31.0	21.5	63.5		50.5	37.0	33.0	23.5		33.0	27.5	25.0

Aug.

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	25.0	24.0	23.5				53.0							72.0	259.0
2	25.0	24.0	23.5				53.0	57.5						72.0	259.0
3	25.0	24.0	24.0				53.0	58.0						72.0	259.0
4	26.0	24.0	24.5				53.0	59.0						72.0	259.0
5	26.0	25.0	25.0				53.0	61.0						72.0	259.0
6	26.5	25.0	25.0				53.0	62.0						72.0	259.0
7	27.0	25.0	25.0				53.0	65.0						72.0	259.0
8	27.0	27.5			61.5	55.5	55.5	71.0						78.0	265.0
9	27.5	27.5			61.0	55.5	55.5							78.0	265.0
10	27.0	27.0			60.0	55.5	55.5							78.0	265.0
11	27.0	27.0			59.0	55.5	55.5							78.0	265.0
12	27.0	26.5			58.5	55.5	55.5							78.0	265.0
13	26.5	26.5			58.0	55.5	55.5							78.0	265.0
14	26.0	26.5			57.0	55.5	55.5						26.0	78.0	265.0
15	26.0	26.5			56.0	55.5	55.5						26.0	78.0	265.0
16	25.5	25.5			56.0	55.5	55.5						26.0	78.0	265.0
17	25.5	25.5			56.0	55.5	55.5						26.0	78.0	265.0
18	25.0	24.5			56.0	55.5	55.5						26.0	78.0	265.0
19	25.0	24.5			56.0	55.5	55.5						26.0	78.0	265.0
20	24.5	24.5			56.0	55.5	55.5						26.0	78.0	265.0
21	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
22	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
23	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
24	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
25	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
26	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
27	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
28	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
29	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
30	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0
31	24.0	24.0			56.0	55.5	55.5						26.0	78.0	265.0

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
275.0		175.5	250.5			87.0		52.0	51.5	57.0	49.0	41.0	42.0	40.5	
274.0		175.5	247.5			85.5		52.0	52.0	57.0	49.0	41.0	43.0	41.0	
274.0		175.5	244.0			85.5		52.5	53.5	57.0	49.0	43.0	43.0	41.5	
274.0		175.5	239.5			85.5		53.0	56.5	57.0	49.0	43.5	43.0	41.5	
274.0		176.0	236.0			85.5		53.0	60.0	57.0	49.0	44.0	43.0	41.5	
275.0		176.0	232.5			85.5		53.5	61.0	57.0	49.0	44.5	43.0	42.0	
275.0		178.0	230.5			85.5		53.5	64.0	57.5	49.0	44.5	43.0	42.0	
160.5	160.5	179.5		83.0	98.0	98.0		53.0	63.5	57.5	49.5	44.0	42.5		
162.0	162.0	179.5		82.0	99.0	99.0		53.0	66.0	58.0	49.5	44.0	42.5		
163.0	163.0	180.0		81.0	97.0		53.0	53.0	66.5	58.0	49.5	44.0	42.5		
164.0	164.0	180.5		80.0	94.5		53.5	53.5	66.0	58.5	49.0	43.5	42.5		
166.0	166.0	180.5		78.5	92.0	53.5	54.0	65.5	65.5	58.5	49.5	43.5	42.5		
167.0	167.0	201.5		78.0	89.0	53.5	55.0	64.5	56.0	47.5	48.0	42.0	42.5		
169.0	169.0	208.5		74.0	87.5	53.5	53.5	63.0	51.0	47.0	48.0	42.5	42.5		
169.5	169.5	204.0		75.0	86.5	53.0	52.5	61.5	52.5	46.0	48.0	42.0	42.5		
170.0	170.0	200.5		77.5	84.0	51.5	52.0	60.5	52.0	46.0	48.0	41.5	42.5		
171.0	171.0	201.5		80.0	83.5	51.0	52.0	59.0	51.5	45.0	48.0	41.5	41.5		
171.5	171.5	244.0		83.0	80.0	53.5	51.5	59.0	51.0	44.5	48.0	41.5	41.0		
171.5	171.5	254.0		85.0	79.0	52.0	50.5	58.0	50.0	43.5	48.0	41.5	40.0		
175.5	175.5	254.5		86.0	78.0	52.0	50.0	57.0	49.5	43.5	48.0	41.5	40.0		
176.5	176.5	270.5		86.0	78.0	52.0	50.0	57.0	49.0	43.5	48.0	41.5	40.0		
176.0	176.0	268.5		87.0	78.0	52.0	50.5	57.0	49.0	43.5	48.0	41.5	40.0		
176.0	176.0	263.0		87.5	78.0	52.0	51.0	57.5	49.0	43.5	48.0	41.5	40.0		
176.5	176.5	256.5		87.0		52.0	51.5	57.5	49.0	43.5	48.0	41.5	40.0		

Sep.

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1		87.5	67.0	60.5	55.0	47.0	44.0	43.0	39.0	35.5	39.0			44.0	33.5
2		86.0	67.0	60.5	54.5	47.0	44.0	43.0	39.0	35.0	42.0			44.0	33.5
3		85.0	67.0	60.5	54.5	47.0	44.0	43.0	39.0	35.0	52.5			44.0	33.5
4		84.0	67.0	60.5	54.5	47.0	44.5	43.0	39.0	36.0	67.0			44.0	33.5
5		83.5	67.0	60.0	53.0	47.5	44.5	43.0	38.5	36.0	83.0			44.0	33.5
6		82.5	67.0	60.0	53.0	47.5	44.5	43.0	38.5	36.5	119.0			44.0	33.5
7		82.0	67.0	60.0	53.0	47.5	45.0	43.0	39.0	36.5	153.0			44.0	33.5
8	142.0	77.0	62.5	55.0	52.0	47.5	45.0	41.0	39.5	37.5			51.5	37.0	33.5
9	139.0	76.5	62.5	54.0	52.0	47.5	45.0	41.0	39.0	37.0			50.5	37.0	33.5
10	138.0	76.0	62.0	52.5	52.0	47.0	45.0	41.0	39.0	37.0			49.0	36.0	33.5
11	132.5	75.5	61.5	53.0	53.0	47.5	45.0	41.0	39.0	37.0			48.0	35.5	33.5
12	129.0	75.0	61.0	53.5	53.0	47.0	45.0	41.0	38.5	37.0			46.5	35.0	32.5
13	127.5	74.0	61.0	53.0	52.0	46.5	44.0	40.0	37.0	37.5			46.0	35.0	31.5
14	126.5	73.5	61.0	52.0	51.0	46.0	43.5	39.5	38.5	34.0			44.0	35.0	31.5
15	123.0	72.5	61.0	53.0	51.0	43.0	43.0	39.5	35.0	33.0			44.0	34.0	30.5
16	119.0	71.5	61.0	53.0	49.0	45.5	43.0	39.0	35.0	33.0			44.0	33.5	29.0
17	113.0	70.5	61.0	59.0	49.0	45.0	43.0	39.5	35.0	33.0			44.0	33.5	29.0
18	106.5	70.0	61.0	59.0	44.0	44.0	43.0	38.0	35.5	25.5			44.0	33.5	29.0
19	101.0	69.0	61.0	60.5	47.5	44.0	42.0	38.5	35.5	25.5			44.0	33.5	29.0
20	96.5	68.5	61.0	60.5	47.5	43.5	42.0	37.0	35.5	25.5			44.0	33.5	29.0
21	93.0	67.5	60.5	59.0	47.0	43.5	42.5	37.0	35.0	25.5			44.0	33.5	29.0
22	91.5	67.5	60.5	58.5	47.0	43.5	43.0	37.0	35.0	23.0			44.0	33.5	29.0
23	90.0	67.0	60.5	57.0	46.0	43.0	43.0	36.0	35.0	27.0			44.0	33.5	29.0
24	88.5	67.0	60.5	56.0	45.5	43.5	43.0	39.0	35.0	33.5			44.0	33.5	29.0

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
29.0	20.5	16.5	17.0	13.0	9.5	80.0	80.0	100.5	115.0	104.0	102.0			134.0
29.0	20.5	16.5	17.0	13.5	9.5	80.0	80.0	101.5	114.0	104.0	103.0			131.5
29.0	20.5	17.0	17.0	13.5	9.5	80.0	80.0	101.5	112.5	104.0	104.0			134.0
29.0	20.5	17.0	17.0	13.5	10.0	80.0	80.0	103.0	112.0	104.0	105.0			133.5
29.0	20.0	17.5	17.0	12.5		80.0	80.0	103.5	111.5	101.0	107.0			132.5
29.0	20.0	17.5	16.5	13.5		80.0	80.0	104.0	111.0	104.0	110.0			132.0
29.0	20.0	18.0	16.5	14.0		80.0	80.0	105.0	110.5	104.0	114.5			131.0
25.5	20.0	18.0	17.0	11.0		80.0	80.0	106.0	110.5	104.0			131.5	130.5
25.5	20.0	18.0	17.0	11.0		80.5	80.5	107.5	109.5	104.0			131.0	129.5
25.5	20.0	17.5	16.5	11.5		100.0	100.0	108.5	109.5	101.0			126.0	129.0
24.5	19.0	17.5	16.5	11.5	90.0	80.0	80.0	109.5	109.5	104.5	101.0		120.0	127.0
24.0	18.5	17.5	16.5	11.0	99.0	80.0	80.0	111.0	109.0	104.0	102.5		127.5	127.5
23.0	18.5	17.0	16.5	11.0	80.0	80.0	80.0	112.0	107.5	103.0			127.0	127.5
22.5	18.0	17.0	16.0	10.0	80.0	80.0	80.0	112.0	107.0	102.0			126.5	126.5
22.0	17.5	17.0	15.0	9.5	80.0	80.0	80.0	112.5	106.5	101.5			126.0	125.0
21.0	17.0	17.0	14.0	9.0	80.0	80.0	80.0	112.5	106.0	101.5			125.0	124.0
20.5	17.0	17.0	13.5	9.0	80.0	80.0	80.0	111.0	105.0	100.5			124.0	123.0
20.0	17.0	17.0	13.0	9.0	80.0	80.0	80.0	111.0	105.0	100.5			123.5	122.5
20.0	16.5	17.0	13.0	9.0	80.0	80.0	80.0	111.0	105.0	100.0			123.0	122.0
20.0	16.0	17.0	13.0	9.0	80.0	80.0	80.0	111.0	105.0	100.0			122.5	121.0
20.0	16.0	17.0	12.5	9.0	80.0	80.0	80.0	111.5	104.5	100.0			122.0	120.0
20.5	16.0	17.0	12.5	9.0	80.5	80.0	80.0	111.5	104.0	100.5			122.0	119.5
20.5	16.5	17.0	13.0	9.0	80.5	80.0	80.0	111.5	104.0	101.0			122.0	119.5

Oct.

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	118.5	115.0	110.0	131.0	113.5	10.2	7.2	5.6	4.1	2.7	1.9	1.7	1.6	6.1	6.4
2	118.5	115.0	112.0	130.0	113.5	10.2	7.2	5.7	4.4	2.7	2.0	1.7	1.6	6.2	6.4
3	118.5	115.0	113.0	129.0	113.5	9.8	7.2	5.7	4.4	2.6	2.0	1.9	1.6	6.0	6.9
4	118.5	114.5	115.0	128.0	114.0	10.2	7.6	6.0	4.4	2.6	2.2	1.9	1.6	6.0	7.7
5	118.5	114.5	121.0	128.0	114.0	10.2	7.6	6.3	4.4	2.5	2.2	1.9	1.6	5.8	7.2
6	118.0	115.0	135.0	127.0	113.5	10.4	7.6	5.8	4.4	2.5	2.2	1.9	1.6	5.7	7.4
7	118.0	115.0	143.0	127.0	112.5	10.4	7.6	5.6	4.4	2.5	2.2	2.2	1.6	5.7	7.8
8	118.0	114.0	151.0	127.0	112.5	10.4	7.6	5.6	4.4	2.5	2.2	2.2	1.6	5.7	8.4
9	117.0	113.0	153.0	120.0	112.0	10.1	7.7	5.8	4.4	2.5	2.2	2.2	1.6	5.3	8.4
10	116.5	113.0	154.0	119.0	107.0	9.6	7.7	5.6	4.3	2.5	2.2	1.7	1.6	5.3	8.8
11	116.5	112.5	152.5	118.5	104.0	9.2	7.2	5.6	4.0	2.5	2.2	1.6	1.5	4.4	9.3
12	116.5	111.5	150.5	118.5	104.0	8.8	7.2	5.6	3.7	2.5	2.2	1.8	1.5	4.4	9.9
13	116.5	111.0	148.0	118.0	104.0	8.8	6.5	5.5	3.3	2.5	2.2	1.8	1.5	4.3	10.6
14	115.5	111.0	145.5	115.0	104.0	8.2	6.5	4.7	3.0	2.2	1.6	1.5	1.5	3.6	9.9
15	115.5	111.0	142.5	114.5	104.0	7.6	6.2	4.6	2.8	1.9	1.6	1.5	1.5	3.6	9.9
16	115.5	110.5	140.0	114.5	104.0	7.2	6.0	4.4	2.4	1.6	1.6	1.5	1.5	3.4	10.2
17	115.5	110.5	138.0	114.0	104.0	7.0	5.7	4.1	2.4	1.6	1.6	1.5	1.5	3.4	10.4
18	115.5	110.0	137.0	114.0	104.0	6.8	5.8	4.0	2.2	1.6	1.6	1.5	1.5	3.4	10.4
19	115.5	110.0	135.5	114.0	104.0	7.0	5.4	4.0	2.0	1.6	1.6	1.6	1.6	3.4	10.4
20	115.5	110.0	134.0	114.0	104.0	7.2	5.4	4.0	2.0	1.6	1.6	1.6	1.6	3.6	10.0
21	115.0	110.0	133.0	114.0	104.0	7.2	5.4	4.0	2.0	1.8	1.6	1.6	1.6	3.6	10.0
22	115.0	110.0	132.0	114.0	10.0	7.2	5.4	4.0	2.4	1.8	1.6	1.6	1.6	3.6	8.8
23	115.0	110.0	132.0	114.0	10.0	7.2	5.4	4.0	2.4	1.8	1.6	1.6	1.6	3.6	8.8
24	115.0	110.0	132.0	114.0	10.0	7.2	5.4	4.0	2.4	1.8	1.6	1.6	1.6	3.6	8.8

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
7.7	6.2	4.2	3.0			6.1	6.1	5.4	21.2	9.4	6.1	4.1	3.4		
7.6	6.2	4.3	3.1			6.2	6.2	5.6	20.7	9.0	6.2	4.3	3.5		
7.4	6.0	4.4	3.4			6.3	6.3	5.4	20.3	8.8	6.3	4.4	3.6		
7.6	6.2	3.8	3.6			6.0	6.0	5.8	19.6	8.7	6.6	4.5	3.6		
7.6	6.4	3.3	3.6			6.1	6.1	6.1	18.8	8.4	6.1	4.4	3.6		
7.5	6.4	3.4	3.7			6.2	6.2	6.3	18.0	8.4	5.9	4.4	3.8		
7.2	6.0	3.0	3.9			6.0	6.0	6.6	17.6	8.6	5.6	4.4	3.8		
7.6	6.0	3.2	3.8			6.8	6.2	7.6	17.0	8.4	5.9	4.4	3.8	11.4	
7.8	6.6	3.6	3.7			9.0	6.0	7.3	17.1	8.5	6.0	4.5	3.8	10.8	
7.8	5.6	3.7	9.0			6.5	6.4	8.4	16.4	8.3	6.1	4.4	3.8	10.7	
7.6	5.8	4.2	8.3			6.4	6.4	9.2	16.0	8.4	5.9	4.4	3.8	10.2	
7.4	5.6	4.8	7.5			5.8	5.8	10.5	15.2	7.7	5.5	4.4	3.8	9.9	
7.1	5.4	4.2	6.0			6.0	6.0	10.2	14.4	7.0	5.2	3.8	3.8	9.5	
6.8	5.2	3.5	5.9			5.9	5.9	11.1	14.3	7.0	5.1	3.8	3.8	9.3	
6.6	5.0	3.3	5.3			5.0	5.0	12.2	13.2	6.7	5.0	3.8	3.5	9.1	
6.4	4.6	4.4	4.8			5.7	5.7	13.1	13.3	6.6	4.5	3.6	3.6	8.8	
6.2	4.4	4.2	4.4			5.4	5.4	13.7	12.1	6.6	4.4	3.5	3.5	8.8	
6.2	4.4	4.2	4.2			5.4	5.4	14.4	11.7	6.6	4.4	3.5	3.5	8.8	
6.3	4.3	4.2	4.2			5.5	5.5	15.0	11.1	6.6	4.4	3.4	3.4	8.8	
6.3	4.3	4.2	4.2			5.5	5.5	16.0	10.1	6.6	4.4	3.4	3.4	8.8	
6.3	4.3	4.2	4.2			5.5	5.5	17.0	10.0	6.6	4.4	3.4	3.4	8.8	
6.3	4.3	4.2	4.2			5.5	5.5	18.0	10.1	6.6	4.4	3.4	3.4	8.8	
6.4	4.3	4.2	4.2			5.5	5.5	19.0	9.5	6.1	3.8	3.1	3.1	8.8	7.5

Nov.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	7.6	5.1	2.6	2.4	1.1	1.4	4.5	4.6	2.2	2.4	1.9	0.0	1.8	11.2	5.5
2	7.7	5.5	2.9	2.4	1.1	1.4	4.6	4.5	2.2	2.4	1.9	0.0	1.7	12.0	5.2
3	7.4	5.5	2.6	2.4	1.1	1.4	4.7	4.5	2.2	2.4	1.9	0.0	1.7	12.4	5.3
4	7.7	5.4	2.2	2.4	1.1	1.4	4.7	4.5	2.2	2.4	1.9	0.0	2.0	12.4	5.3
5	7.0	5.5	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	2.3	12.0	5.0
6	7.2	5.5	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	2.2	12.0	5.3
7	7.4	5.5	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	4.2	12.2	5.2
8	7.9	5.5	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	6.0	12.4	5.0
9	7.1	5.5	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	7.6	12.4	5.0
10	7.0	5.2	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	9.9	12.4	5.0
11	6.6	5.0	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	12.3	12.3	5.4
12	6.2	4.8	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	18.0	16.0	4.6
13	6.6	4.7	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	18.8	18.8	4.4
14	6.6	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	20.4	20.4	4.5
15	6.5	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	20.5	20.7	4.4
16	6.5	4.6	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	20.4	20.4	4.4
17	6.5	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	19.7	19.7	4.4
18	6.5	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	19.2	19.2	4.4
19	6.5	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	19.1	19.1	4.4
20	6.5	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	19.1	19.1	4.4
21	6.5	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	19.1	19.1	4.4
22	6.5	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	19.1	19.1	4.4
23	6.5	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	19.1	19.1	4.4
24	6.5	4.4	2.2	2.4	1.1	1.4	4.8	4.5	2.2	2.4	1.9	0.0	19.1	19.1	4.4

[illegible]

Dec.

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	13.4			9.4	7.8	6.8	8.4	8.3	5.8	6.0	6.4	6.5			3.2
2	13.2			9.9	7.9	7.0	8.4	7.8	6.6	6.1	6.4	6.5			3.7
3	13.8			9.9	8.1	7.1	9.4	7.8	6.6	6.0	6.6	6.3			2.1
4	12.2			8.8	7.7	7.1	9.9	7.8	6.6	6.0	6.6	6.3			3.0
5	12.4			8.8	7.7	7.2	10.6	8.8	6.6	6.0	6.6	6.4			3.0
6	12.6			8.8	7.7	7.2	11.1	8.8	6.6	6.0	6.8	6.6		4.4	2.9
7	11.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	6.8	6.6		4.4	3.0
8	11.3			8.8	7.7	7.0	11.5	8.8	6.6	6.0	7.0	6.6		4.4	3.2
9	11.0			8.8	7.7	7.2	11.8	8.8	6.6	6.0	7.5	6.6		4.4	3.2
10	10.8			8.8	7.7	7.2	12.1	8.8	6.6	6.0	7.6	6.6		4.4	3.2
11	10.8			8.8	7.7	7.0	11.7	8.8	6.6	6.0	7.8	6.6		4.4	3.0
12	10.8			8.8	7.7	7.1	11.2	8.8	6.6	6.0	7.8	6.6		4.4	3.0
13	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
14	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
15	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
16	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
17	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
18	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
19	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
20	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
21	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
22	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
23	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0
24	10.8			8.8	7.7	7.0	11.4	8.8	6.6	6.0	7.8	6.6		4.4	3.0

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
2.1	2.2	1.6	1.6	7.7	7.4	7.8	14.1	1.6	18.8	12.0	11.8	17.0	9.5	42.0	41.5
2.2	2.3	1.6	1.6	7.7	7.0	7.7	22.8	1.6	16.4	11.8	11.3	16.3	9.3	41.5	41.3
2.3	2.4	1.6	1.6	7.7	7.0	7.7	39.2	1.6	14.7	11.3	11.3	15.3	9.0	41.3	41.3
2.4	2.5	1.6	1.6	7.7	7.0	7.7	48.9	1.6	13.1	11.3	11.0	14.5	9.0	41.3	41.0
2.5	2.6	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.3	11.0	13.8	8.8	41.3	40.8
2.6	2.7	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.3	10.8	13.5	8.5	41.3	40.8
2.7	2.8	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.3	10.8	12.8	8.3	41.3	40.8
2.8	2.9	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.3	10.8	12.5	8.5	41.3	40.8
2.9	3.0	1.6	1.6	7.7	7.0	7.7	48.8	1.6		12.3	11.0	12.0	8.0	40.3	40.8
3.0	3.1	1.6	1.6	7.7	7.0	7.7	48.8	1.6		12.0	11.5	12.0	7.8	40.3	40.8
3.1	3.2	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.5	12.0	11.5	7.3	40.0	40.5
3.2	3.3	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.0	12.5	11.5	7.3	40.0	40.0
3.3	3.4	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.3	13.5	11.0	7.0	40.0	40.0
3.4	3.5	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.0	14.8	11.0	7.5	40.3	40.0
3.5	3.6	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.5	15.5	11.0	7.7	40.5	40.0
3.6	3.7	1.6	1.6	7.7	7.0	7.7	48.8	1.6		11.0	17.3	11.0	7.8	40.8	40.0
3.7	3.8	1.6	1.6	7.7	7.0	7.7	48.8	1.6		12.3	18.5	10.8	8.3	41.0	40.3
3.8	3.9	1.6	1.6	7.7	7.0	7.7	48.8	1.6		12.0	19.3	10.0	7.7	41.0	40.0
3.9	4.0	1.6	1.6	7.7	7.0	7.7	48.8	1.6		12.0	20.3	10.0	7.7	41.0	39.3
4.0	4.1	1.6	1.6	7.7	7.0	7.7	48.8	1.6		12.0	21.0	9.5	7.5	41.5	39.0

Feb.

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	26.0	23.3	24.8	27.8	30.3	31.3	29.8	19.0	18.0	25.5	24.3	24.8	62.5	44.5	34.8
2	25.8	23.3	24.8	28.0	29.8	31.3	29.0	19.0	18.0	27.0	24.3	25.8	63.8	43.3	34.8
3	25.3	23.3	24.8	28.0	29.8	31.3		19.0	18.0	27.0	24.3	25.8	65.5	41.8	33.8
4	25.0	22.8	25.0	27.8	29.3	31.0		19.0	17.8	30.3	23.0	30.0	66.3	41.0	33.8
5	25.0	22.8	24.8	27.5	29.3	31.0		19.0	17.5	31.0	23.0	33.5	67.3	40.0	33.8
6	25.3	21.8	24.8	27.3	29.0	31.3		19.0	17.0	32.3	23.0	37.5	67.3	39.5	32.8
7	25.3	21.8	24.8	27.0	28.8	31.3		19.0	17.0	33.0	23.0	42.0	66.0	37.8	32.8
8	25.5	21.8	24.3	27.0	28.3	31.3		17.3	16.5	33.3	23.0	49.0	64.0	36.8	32.8
9	25.3	21.8	24.3	27.0	28.3	31.3		17.3	16.5	33.3	23.0	57.0	62.0	36.0	31.8
10	25.3	20.8	23.8	26.5	27.5	31.3		17.0	16.3	33.3	23.0	64.0	54.0	34.5	31.8
11	20.3	20.8	23.8	26.0	27.5	31.0		17.0	16.0	32.8	23.0	68.0	51.0	33.8	30.8
12	20.3	20.8	23.5	25.5	27.5	31.0		17.3	16.0	31.8	23.0	70.0	51.0	32.5	29.8
13	20.0	20.8	23.5	25.5	27.5	31.0		17.0	16.0	31.0	22.8	71.3	51.5	32.8	29.8
14	20.0	20.8	23.5	25.3	27.8	30.8		16.8	16.0	30.0	22.0	70.0	51.8	32.5	29.5
15	20.0	20.8	23.8	25.5	27.8	30.5		16.5	15.8	28.8	21.8	69.3	52.5	32.8	29.8
16	20.0	21.8	24.0	25.8	28.0	30.5		16.5	16.0	28.3	21.8	67.8	53.8	32.0	29.5
17	21.0	21.8	24.0	26.0	28.8	30.8		17.0	16.8	27.3	21.8	65.8	54.8	32.8	30.0
18	21.0	22.8	25.5	27.8	29.5	30.8		17.3	17.8	27.0	22.0	63.3	54.5	33.0	31.0
19	21.0	22.8	25.5	27.8	29.5	30.8		18.0	18.3	26.5		61.3	53.8	34.3	31.0
20	21.0	23.0	25.8	29.3	30.8	30.0		18.3	19.0	26.3	22.5	61.3	52.5	34.8	31.3
21	22.0	23.0	25.8	29.3	30.8	29.3		20.0	20.5	25.5	22.8	60.0	51.3	34.8	31.3
22	22.0	23.0	25.8	30.0	30.8	29.3		20.5	20.5	25.3	23.3	59.8	50.0	34.8	31.0
23	23.0	24.0	27.3	30.3	31.0	29.5	19.0	18.3	22.3	25.0	23.5	60.3	48.3	35.0	30.8
24	23.0	24.0	27.3	30.3	31.0	29.0	18.8	18.3	22.8	24.8	23.8	61.3	47.0	35.0	30.8

16	17	18	19	20	21	22	23	24	25	26	27	28
27.8	26.3	67.0	36.5	119.0	73.5	39.0	30.3	28.5	32.8	25.8	25.0	36.0
27.8	26.3	65.0	36.0	113.8	72.3	39.0	30.3	28.3	32.5	25.8	25.0	35.8
27.5	28.0	63.8	36.0	110.5	72.0	39.0	30.0	28.0	32.0	25.8	25.0	35.3
27.3	28.0	61.5	36.0	107.0	70.5	39.2	30.0	29.0	31.8	25.8	25.0	35.0
26.5	28.0	59.3	36.0	103.5	70.0	39.5	30.0	29.3	31.5	25.8	25.3	35.3
26.5	28.8	58.0	36.3	100.3	69.8	39.3	29.8	30.3	31.0	25.8	25.5	35.3
26.5	29.3	57.3	36.3	97.3	68.8	39.0	29.5	31.0	30.5	26.3	25.8	35.8
26.3	30.3	55.3	36.5	95.5	68.8	38.8	29.3	31.8	30.0	26.5	25.8	36.3
26.5	32.0	55.5	36.8	95.5	67.8	38.8	29.3	32.0	29.5	26.3	25.8	37.0
26.5	33.3	53.3	35.3	89.0	67.0	38.8	29.0	31.8	28.8	26.0	25.8	37.8
26.5	34.3	44.8	36.3	88.5	53.8	37.8	27.8	31.8	28.3	26.0	24.8	39.0
25.0	37.3	42.3	44.3	45.3	52.5	37.3	27.3	32.3	28.0	25.8	26.8	39.8
24.0	42.3	41.0	41.3	46.0	51.3	36.6	27.0	32.3	27.8	25.8	26.8	40.8
24.0	44.0	40.3	40.3	45.3	51.3	36.6	27.0	32.3	27.3	25.8	26.8	42.0
24.3	63.3	40.3	57.0	84.5	50.3	35.5	27.0	33.3	28.0	26.0	27.8	42.0
24.3	70.8	40.3	63.3	87.3	50.3	35.5	27.0	33.3	28.0	26.0	27.8	41.8
25.0	76.0	39.3	76.3	89.3	50.0	35.3	26.8	33.3	28.0	26.0	30.8	41.8
25.8	77.3	39.5	87.3	91.5	49.3	35.3	26.8	33.3	28.0	26.0	31.3	40.8
25.8	76.8	38.8	97.0	91.5	49.0	32.0	26.8	33.3	28.0	25.8	33.1	39.0
25.8	74.0	38.0	105.3	88.8	48.8	31.8	26.8	33.3	28.0	25.8	34.8	38.8
25.8	74.0	37.6	112.0	77.5	48.8	31.3	26.8	33.3	28.0	25.8	35.5	37.0
25.8	70.5	37.3	116.8	76.0		31.0	26.8	33.3	28.0	25.5	35.8	36.3
25.8	69.0	37.0	118.8	74.8	39.0	30.8	26.8	33.0	27.8	25.0	36.0	35.8

Apr.

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	24.8	21.0	18.8	18.0	53.0	78.0	44.3	34.0	29.5	56.3	42.0	28.8	25.0		33.8
2	24.8	20.8	18.8	18.8	65.0	77.0	43.5	34.0	29.0	55.0	41.3	28.5	25.3		32.8
3	24.8	20.8	18.8	20.8	78.0	75.3	43.5	34.0	29.0	54.3	41.3	28.0	25.8		32.0
4	24.8	20.8	18.8	22.8	94.0	73.3	43.0	33.8	29.0	53.0	40.8	28.0	26.8		31.0
5	24.8	20.5	18.8	24.0	105.5	72.0	42.5	33.5	30.8	52.0	40.3	28.0	26.8		29.3
6	24.8	20.3	18.8	27.0	112.0	70.5	42.3	33.0	30.8	50.5	39.8	28.3	26.8		28.5
7	24.8	20.3	18.8	30.5	114.8	69.0	42.0	32.5	30.8	49.5	38.0	28.8	26.8		28.5
8	24.5	20.0	18.0	35.8	115.8	68.0	41.5	32.5	29.5	44.5	38.0	28.8	26.8		28.5
9	24.5	20.0	18.0	40.3	115.8	61.3	40.8	32.5	29.5	43.5	37.3	28.5	26.8		28.5
10	24.5	20.3	18.3	45.8	113.5	60.5	40.5	32.5	30.3	42.5	37.3	28.5	26.8		28.5
11	24.5	20.3	18.3	48.8	111.0	59.3	40.3	32.3	32.3	41.3	36.5	28.5	26.8		28.5
12	24.5	20.0	18.5	51.5	106.5	57.5	40.0	32.0	35.5	41.3	36.3	28.3	26.8		28.5
13	24.5	20.0	18.8	52.8	102.0	55.5	39.5	32.0	40.0	41.0	31.0	28.3	26.8		28.5
14	24.5	20.3	19.0	53.0	98.5	55.3	39.3	32.0	44.8	40.8	30.8	28.3	26.8		28.5
15	24.5	20.3	19.3	53.3	94.5	54.0	39.0	32.0	50.0	41.0	30.3	28.0	26.8		28.5
16	24.0	20.3	19.5	52.5	90.5	52.5	38.5	32.0	56.0	41.3	34.5	28.0	26.8		28.5
17	24.0	20.3	19.8	50.5	87.5	51.3	38.3	32.0	61.3	41.5	34.3	28.0	26.8		28.5
18	23.3	20.3	19.8	48.5	87.3	49.8	38.0	32.0	64.0	42.3	33.5	27.5	26.8		28.5
19	23.3	20.3	19.8	46.5	84.3	48.8	37.8	31.5	65.0	43.0	32.5	27.0	26.8		28.5
20	22.8	20.3	19.8	45.0	83.0	48.0	37.0	30.8	64.5	43.5	31.5	26.8	26.8		28.5
21	22.3	19.8	18.8	43.8	82.5	46.5	36.5	30.5	63.3	43.5	30.5	26.3	26.8		28.5
22	22.0	19.5	18.8	43.5	81.8	45.5	36.0	30.0	61.3	43.0	30.0	26.5	26.8		28.5
23	22.0	19.0	18.8	43.8	80.3	45.5	35.0	29.5	60.0	42.5	29.3	26.0	26.8		28.5
24	22.0	18.8	18.3	46.5	79.3	44.5	34.5	29.5	58.3	42.3	28.3	25.0	26.8		28.5

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
11.5	4.3	17.0	19.3	14.8	10.3	4.3	4.5	13.3	9.5	4.3	4.5	4.3	23.3	23.3
10.5	4.3	17.0	17.8	14.5	10.3	4.3	4.5	13.5	9.5	4.5	4.5	4.3	23.3	23.3
10.3	4.3	17.5	17.0	14.5	10.3	4.3	4.5	14.0	9.8	5.0	4.5	4.3	23.5	23.5
9.8	4.3	18.3	16.8	14.5	10.3	6.8	4.5	14.5	10.0	5.3	4.5	4.3	24.0	24.0
9.8	4.3	19.5	17.0	14.8	10.3	8.0	4.5	14.8	10.0	6.0	4.5	4.3	24.5	24.5
9.5	4.0	22.0	17.0	15.0	10.3	8.3	4.5	14.8	10.5	6.3	4.8	4.3	25.0	25.0
9.8	4.3	23.5	17.3	15.3	10.5	8.0	4.5	15.3	10.8	6.5	5.0	4.3	25.3	25.3
9.8	4.3	24.0	17.3	15.5	4.8	7.5	4.0	15.8	10.8	6.8	5.0	4.3	25.3	25.3
9.0	4.0	24.3	17.3	15.8	5.3	7.3	20.0	16.0	11.5	7.3	5.3	20.5	20.5	26.0
8.8	19.3	24.5	17.3	15.5	5.3	7.3	20.0	15.8	11.8	7.8	5.5	21.0	21.0	26.5
8.5	19.8	24.5	17.5	15.5	5.5	6.8	18.3	15.3	11.3	7.3	5.8	21.8	21.8	27.0
8.3	19.8	24.5	18.0	14.3	5.5	6.3	17.8	15.0	11.5	7.5	5.8	22.0	22.0	27.5
8.3	19.8	24.3	18.3	13.0	5.0	6.3	17.3	15.0	11.0	7.0	5.3	22.5	22.5	27.8
8.0	19.3	23.8	18.3	12.0	4.8	6.3	17.3	15.0	11.0	7.0	5.3	23.0	23.0	27.8
7.5	19.5	23.8	18.3	11.5	4.5	6.3	17.3	14.8	11.0	7.0	5.0	23.3	23.3	27.8
7.8	19.5	23.0	18.3	11.5	4.5	6.3	17.3	14.8	10.8	6.8	4.0	24.5	24.5	26.5
6.8	19.3	22.8	18.3	12.0	4.5	6.3	17.3	14.3	10.3	6.3	4.0	24.8	24.8	26.3
5.0	19.0	22.8	18.8	11.0	4.5	6.3	17.3	13.5	8.5	6.0	4.0	24.3	24.3	25.5
4.3	19.0	22.0	17.8	10.0	4.3	6.3	16.0	12.5	6.0	5.8	4.0	23.8	23.8	24.8
4.3	18.5	21.5	16.8	10.0	4.3	6.3	15.0	11.5	4.3	5.3	4.0	23.5	23.5	24.0
4.3	18.0	21.3	16.0	10.3	4.3	6.3	14.5	11.0	4.3	4.3	4.0	23.3	23.3	23.5
4.3	17.5	20.8	15.5	10.3	4.3	6.3	13.5	10.0	4.3	4.5	4.3	23.5	23.5	22.5
4.3	17.0	20.3	15.0	10.3	4.3	6.3	13.5	9.5	4.3	4.5	4.3	23.5	23.5	21.8

May

Day Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	21.3	20.0	21.0		7.5	7.0	16.5	9.5	6.6	139.5	4.3	8.0	7.3	6.8	4.3
2	21.0	20.0	21.3		7.5	7.0	16.3	9.3	6.6	146.3	4.3	8.0	7.0	6.0	4.3
3	20.8	20.3	22.0		7.5	7.0	16.3	9.3	6.6	9.3	4.3	8.0	7.0	5.5	4.3
4	21.0	20.3	22.5		7.5	7.0	16.3	9.3	6.6	9.3	4.3	8.0	7.3	5.8	4.3
5	21.3	20.8	23.5		7.5	7.0	16.3	9.3	6.6	9.3	4.3	8.0	7.5	5.5	4.3
6	21.5	21.0	24.5		7.5	7.0	16.5	9.3	6.6	9.3	4.3	8.0	8.0	5.5	4.3
7	21.5	21.3	25.0	21.0	7.5	7.0	16.5	9.3	6.6	9.3	4.3	8.0	8.0	5.5	4.3
8	22.0	21.8	26.3	18.0		19.0	16.5	9.0	4.4	9.3	2.1	9.3	9.3	5.5	4.3
9	22.5	22.2	27.0	15.0	19.5	19.0	16.5	9.0	16.3	9.3	23.0	9.3	9.3	5.5	20.0
10	22.5	22.2	29.0	15.0	18.5	19.5	16.5	9.0	16.3	9.3	24.0	9.3	10.0	19.5	
11	22.8	22.2	32.0	7.5	17.5	19.3	16.5	9.0	16.3	9.3	24.0	9.3	10.3	20.0	
12	23.0	22.2	32.0		16.5	18.0	16.5	9.0	16.3	9.3	25.0	9.3	11.3	20.0	
13	23.0	22.2	32.0	105.0	9.5	18.0	16.5	9.0	16.3	9.3	24.0	9.3	12.0	19.3	
14	23.0	22.2	32.0		7.5	17.3	15.3	9.0	16.3	9.3	23.0	9.3	12.3	18.5	
15	23.0	22.2	32.0		7.5	17.3	14.3	9.0	16.3	9.3	21.0	9.3	12.5	17.5	
16	23.0	22.2	32.0		7.5	17.3	13.3	9.0	16.3	9.3	18.0	9.3	12.8	17.0	
17	23.0	22.2	32.0		7.5	17.3	12.3	9.0	16.3	9.3	15.8	9.3	13.0	16.5	
18	23.0	22.2	32.0		7.5	17.3	12.3	9.0	16.3	9.3	13.5	9.3	13.0	16.5	
19	23.0	22.2	32.0		7.5	17.3	12.3	9.0	16.3	9.3	11.5	9.3	12.5	16.0	
20	22.8	22.2	32.0		7.5	17.3	12.0	9.0	16.3	9.3	8.3	9.3	12.5	16.0	
21	22.5	22.2	32.0		7.5	17.3	11.5	9.0	16.3	9.3	8.3	9.3	11.5	16.0	
22	21.5	22.2	32.0		7.5	17.0	11.0	9.0	16.3	9.3	8.0	9.3	10.5	16.0	
23	20.5	21.5	32.0		7.5	16.8	10.5	9.0	16.3	9.3	8.0	9.3	9.0	15.5	
24	20.3	21.5	32.0		7.5	16.5	9.8	9.0	16.3	9.3	8.0	7.5	7.5	15.0	

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
14.8	50.0	4.3	4.3	4.0	11.3	8.0	74.3	16.5	14.5	41.0	69.5	27.8	16.8		14.8
14.8	50.0	4.3	4.0	4.0	11.3	8.0	73.5	16.3	15.3	39.5	69.3	26.8	16.5		14.8
14.8	50.0	4.3	4.0	4.0	11.3	8.0	72.0	16.0	15.3	38.0	67.0	25.8	16.3		14.8
14.8	122.3	4.3	4.0	4.0	11.5	8.5	70.0	15.8	21.8	37.3	65.5	25.0	16.0		15.0
14.8	147.3	4.3	4.0	4.0	11.5	9.0	66.5	15.5	27.5	36.5	63.5	25.0	16.0		15.0
15.8		4.0	4.0	4.0	11.5	9.3	63.5	15.5	34.3	36.5	62.0	25.0	16.0		15.3
15.8		4.0	4.0	4.0	11.5	9.8	61.0	15.5	42.5	37.0	60.3	24.8	16.3		15.5
16.0		20.0	4.5	19.5	12.0	7.3	58.3	16.0	50.0	39.0	59.3	24.3	16.5		15.5
16.3	72.5	13.3	19.0	19.0	12.0	7.8	53.5	15.3	59.5	40.0	58.3	24.5	16.5	5.0	15.8
16.3	92.8	5.5	18.0	18.8	12.0	8.0	50.0	15.3	66.0	41.5	56.5	24.3	16.5	5.0	16.3
16.3	104.0	4.5	16.0	18.3	12.3	8.3	46.8	15.3	68.8	43.8	55.5	24.0	16.5	20.0	17.0
16.3	107.5	4.5	13.3	17.5	12.5	8.5	45.0	15.3	69.0	45.8	54.0	23.8	16.5	19.5	17.3
16.3	102.0	4.5	11.0	17.3	12.5	8.5	43.0	15.3	69.3	46.5	52.3	23.3	16.5	20.0	17.0
16.0	90.0	4.5	9.5	16.5	12.3	9.0	40.3	14.5	69.3	47.5	50.3	22.5	15.5	17.0	
16.0		4.5	8.5	15.8	12.0	9.8	37.3	14.3	68.5	48.3	49.3	21.5	14.0	16.8	
16.0		4.5	8.5	15.8	12.0	14.5	34.5	13.3	68.0	49.3	48.3	21.5	13.3	16.3	
16.0		4.5	7.7	15.0	11.5	20.0	31.5	13.3	64.0	51.8	46.3	21.0	12.3	16.3	
16.0		4.5	6.3	14.0	10.5	28.0	28.0	13.3	56.3	53.3	44.3	20.5	11.8	16.3	
16.0		4.5	5.5	13.3	10.0	48.0	24.0	13.3	53.3	62.5	42.5	19.5	11.5	15.3	
16.0		4.5	5.0	12.3	9.5	56.5	22.3	13.3	49.5	63.3		18.8	11.5	14.8	
16.0		4.5	4.3	12.3	9.0	63.0	21.3	13.3	47.5	63.3		18.3	11.5	14.8	
16.0		4.5	4.3	11.3	8.5	67.8	19.0	13.3	44.5	70.0		17.5	11.0	14.8	
16.0		4.3	4.3	11.5	8.0	72.0	17.5	13.5	42.5	70.3	29.5	17.3	11.0	14.8	